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COMPARISON OF PROTONS AND ELECTRONS IN THE EXCITATION OF X-RAYS BY IMPACT.

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ABSTRACT.

THE experimental investigation of the laws governing the excitation of x-rays by electron impact appears to be near completion. However, the existence of the proton, with its mass of 1,846 times that of the electron, provides the possibility of introducing a new variable—the mass—into collision experiments. Previous investigations have led to inconclusive results except that it is known that when fast α-particles fall on an element, its characteristic x-ray spectrum is excited.

In the writer's experiments, a mass spectrograph is used to bring either protons or electrons, of same energy in the range from 15 kv. to 25 kv., onto a copper target. The arrangement eliminates several known sources of error. X-radiation produced passed into an argon filled Geiger counter. The procedure was to compare two currents: (1) the current composing an electron beam just intense enough to produce detectable x-rays; and (2) that composing the strongest proton beam which certainly produced less radiation. The ratio of these currents is a minimum value for ρ, the ratio of the efficiencies for excitation of electrons and protons respectively.

The results were that (1) no radiation from proton impacts was observed, (2) the minimum value of ρ from three sets of experiments was 740; 237,000; and 6,400. In view of the directness of the experiment, the order of magnitude of the highest figure is presented as the result, i.e. $\rho = 10^5$. This may be compared with $(M/m) = 1{,}846$, $(M/m)^{3/2} = 79{,}310$, $(M/m)^2 = 3{,}408{,}000$.

Wave mechanical considerations of impact processes indicate that radiation should be produced by proton impacts of energy greater than the critical energy (for copper, 8.86 kv.) but intensity formulas have not been derived. If they can be derived the present results offer the possibility of checking the wave mechanics of impact processes.

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INTRODUCTION.

Since the discovery of x-rays by Roentgen in 1895 there have been conducted a very large number of researches whose purpose was to gain a knowledge of the laws governing the production of such rays. It was learned that the impact of fast electrons upon dense targets was the principal phenomenon having as an accompaniment the emission of x-radiations. With the improvement of vacuum and thermionic technique, it became possible to carry through accurate and reliable experiments which have led to the discovery of empirical laws governing the nature, wave-length and intensity of radiation as a function of the electron velocity and the material of the target.[1] Such experiments are still in progress, but it may now be said with fair certainty that, aside from the secondary considerations introduced by the finite thickness and density of practical targets, it is only necessary to know the velocity of the electrons and of what atoms the target is made to determine all the properties of the radiation.

At first sight, it might be maintained that after electrons of all velocities have been directed against targets of every kind of atom and the properties of the resulting x-rays measured, the experimental investigation of the mechanism of impact and emission will have been completed. It is necessary, however, to remember that classically an electron is merely a charged massive particle, that there is another fundamental charged massive particle—the proton—available for experiments, and that, under the classical laws of electromagnetism, such a particle should radiate and interact with other charges in a way not fundamentally different from the way that an electron does. Although inadequate today, the classical laws have still a suggestive authority, and, on this basis, the existence of the proton may be regarded as furnishing the possibility of changing the mass of the striking particle. A new variable, the mass, is thereby introduced into the impact process and the knowledge of the mechanism of the process nust necessarily be augmented when it is learned what the

[1] Reference may be made to any text on X-Rays; especially: M. Siegbahn, "The Spectroscopy of X-Rays," Chaps. IV and VI (Eng. Ed. Oxford Univ. Press, 1925); and A. H. Compton, "X-Rays and Electrons," Chaps. II and XII (Van Nostrand, 1926).

effect of the altered mass is. At the present time, such a knowledge is particularly desirable, not only because of the importance of the phenomenon itself, but because of the connection of the latter with the new quantum mechanics. From the quantum theoretic standpoint, rapid progress is being made in the effort to describe collisions of electrons with atoms.[2] Success in this effort will almost surely carry with it the possibility of predicting the effect of collisions of protons with atoms. The experiments to be described in this paper constitute the beginning of an attempt to compare the radiation, if any, emitted when a target is struck by fast protons with that emitted when, as usual, electrons are used instead.

Early experiments of this sort have usually been confused by the presence of various spurious effects which destroyed the possibility of unambiguous conclusions. In a very interesting investigation carried out by Thomson,[3] it seemed at first that all such effects had been excluded. His apparatus was arranged to direct hydrogen canal rays onto a target. A side arm carried a photographic plate which could be exposed to the target and so detect radiation from the latter. Between the target and plate was a condenser whose cross field stopped any reflected charged canal rays which would otherwise have produced a spurious image on the plate. He found that when the canal rays were striking the target the plate was blackened. When the rays were stopped by another condenser between the canal and the target no such blackening occurred. This showed that the blackening was truly due to the canal rays striking the target and not to stray light from the discharge tube. The insertion of even the thinnest films of collodium which he could make in front of the plate stopped the blackening completely. He concluded from this that the radiation consisted of very easily absorbed x-rays.

More recently, however, Gerthsen[4] has made a careful study of the impact of hydrogen canal rays on metal targets, and has discovered that many of the reflected rays have lost their charge. Since uncharged canal rays would not have been

[2] M. Born, *Zeits f. Phys.*, *38*, 803 (1926). *Gottingen Nachr.*, p. 146 (1926). W. Elsasser, *Zeits. f. Phys.*, *45*, 522 (1927).

[3] J. J. Thomson, *Phil. Mag.*, *28*, 620 (1914).

[4] C. Gerthsen, *Ann. d. Phys.*, *85*, 881 (1928).

stopped by passage through a condenser, it is possible to ascribe the plate blackening observed by Thomson to their action. Further experiments by Gerthsen dealing with the absorption coefficient of celluloid for such rays added weight to this criticism. Gerthsen then undertook the difficult problem of distinguishing whether or not there was any electromagnetic radiation in the soft x-ray region coming from the target along with the fast neutral particles. Since the absorbing power of celluloid is about the same for each, he resorted to specular reflection at a metal mirror under the assumption that only the electromagnetic radiations would be so reflected. The observations revealed the presence of a weak specularly reflected radiation. The effect was so weak, however, and the assumption involved so uncertain that one hesitates to accept the result finally without further confirmatory experiments.

Both Thomson and Gerthsen were apparently led by theoretical arguments to confine themselves to a search for soft, rather than penetrating, x-rays. The production at the target of ordinary penetrating x-rays would probably have been overlooked unless they had been so strong that they would certainly have been discovered incidentally long since. The results of their investigations did not preclude, therefore the possibility looked for in the writer's experiments, that very weak penetrating x-rays are excited by the impact of fast protons on a target. Whether or not protons of a few thousand volts equivalent kinetic energy can excite ordinary x-rays, it is extremely probable that faster ones could. This argument is based on the many experimental observations of x-radiation due to the impact of α-particles.[5,6] The radiation observed has always been very weak. In the most satisfactory experiments of this kind, those of Bothe and Franz,[6] it was shown that the characteristic radiation (K or L) of the target material was excited without the usual continuous background. The inten-

[5] J. Chadwick, *Phil. Mag.*, *24*, 594 (1912); *25*, 193 (1913). E. Rutherford and H. Richardson, *Phil. Mag.*, *25*, 722 (1913). J. Chadwick and A. S. Russell, *Proc. Roy. Soc.*, *88*, 217 (1913). A. S. Russell and J. Chadwick, *Phil. Mag.*, *27*, 112 (1914). F. P. Slater, *Phil. Mag.*, *42*, 904 (1921). W. Bothe and H. Franz, *Zeits. f. Phys.*, *49*, 1 (1928).

[6] W. Bothe and H. Franz, *Zeits. f. Phys.*, *52*, 466 (1929).

sity of the radiation fell off rapidly with the energy of the α-particles, but in such a way that no evidence was given of the existence of a critical minimum excitation energy before the point was reached where the radiation was too weak for observation. Thus, the possibility remains that α-particles—or, analogously, protons—of energy in the range of 15 to 25 kv. investigated in the writer's experiments might still excite radiation, although it would be very weak. Perhaps compensating this weakness is the feasibility of directing at a target many times as many canal ray protons as α-particles in the experiments referred to.

PRELIMINARY EXPERIMENTS.

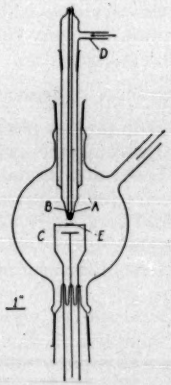
For the purposes of the present research it was necessary to solve, as far as possible, three problems of technique. There were required: first, an abundant source of protons; second, a sensitive detector for x-rays; and, third, a means of eliminating the chance of any spurious radiation arising at the target and reaching the detector. Of these problems, the first presented the greatest difficulty, and only the last was completely solved.

Source of Protons: It would be very desirable to have a source of protons something like the hot cathodes used to furnish electron streams in a vacuum. Such a source was found and used by Dempster.[7] The writer, after some experiments with this type of source in collaboration with Johnson and Bramley, concluded that the rate of production of protons was too small for the purpose at hand. However, a brief record of these preliminary experiments should be made in this place. Fig. 1 is a diagram of the tube used. The conical thimble A was made of platinum .001″ thick. Its tip could be heated by an interior element B consisting of a tungsten spiral imbedded in alundum cement. Temperatures as high as 1000° C. could be maintained and were measured by means of an optical pyrometer. The other electrodes were an ordinary tungsten filament E and a collector plate. Hydrogen could be admitted at any pressure to the inside of the platinum thimble and was observed to diffuse through the platinum at a rate which increased with the temperature. The bulb itself was

[7] A. J. Dempster, *Phil. Mag.*, *3*, 115 (1927).

connected to diffusion pumps through the usual liquid air traps. It was thought that possibly the hydrogen would emerge from the platinum surface in a dissociated and ionized state. However, even with differences of potential up to 500 volts, no ions could be drawn from the thimble to the plate. Next there was applied to the platinum a coating of acid

FIG. 1.



Apparatus for testing various possible thermionic sources of protons.

aluminum phosphate, as suggested by Dempster,[8] and the experiment repeated. Only a small ion current to the plate was observed. Then both experiments were carried through again with the additional condition that a heavy electron stream from E bombarded the surface at A. Again the ion current to the plate was small for the present purposes. In all cases, this current was measured after a transient current, which appeared at the first heating of the thimble after evacuation, died down. The transient current was doubtless due to the emission of ionized traces of alkali metals, a well known phenomenon.[9] The final currents were smaller than 10^{-8}

[8] Private communication.

[9] O. W. Richardson, "The Emission of Electricity from Hot Bodies," Chaps. VI and VII (Longman's, Green and Co.).

amps. from a surface of about 1 cm.^{2} and were by no means certainly composed of protons. It was therefore decided to resort to the older technique of canal rays.

A series of tubes of the canal ray type were built trying different methods of producing a fast proton beam of great intensity. A full description of these tubes would not be worth the space it would require. As a result of these preliminary trials it was learned that the ions could not be accelerated satisfactorily in the discharge which produced them. The velocities were too inhomogeneous and corresponded to too small a fraction of the applied voltage. On the other hand, acceleration in the space behind the canal was quite feasible provided this space was rapidly pumped out. It was also learned that great precautions were necessary against the emission and acceleration of secondary electrons arising wherever the fast ions struck. Such electrons not only tended to start a discharge in the acceleration space, but they, in turn, struck parts of the tube and produced x-rays not readily to be distinguished from those due to the positive impacts. It was also found that ultra-violet light, and possibly soft x-rays, came through the canal from the discharge tube, and, striking the target, not only produced undesirable photo-electrosn, but, by reflection and scattering, constituted another spurious source of radiation. As a result of the experience with these tubes it was possible to design a final tube from which all of the above mentioned difficulties were eliminated.

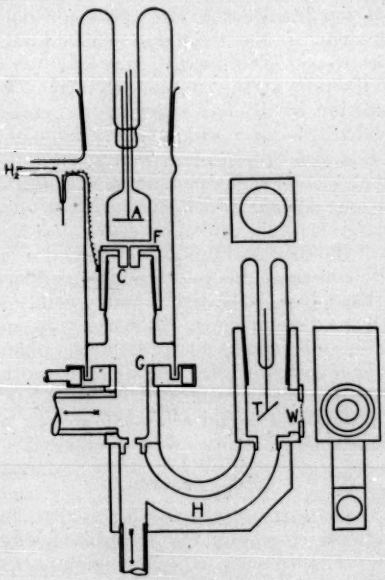
FINAL APPARATUS.

The experimental tube finally used was that illustrated in Fig. 2. It closely resembles the Dempster design of mass spectrograph. The lower or mass analyzing part of the tube was of brass, and the upper part, where the ions were produced and accelerated, of glass. The glass parts were cemented to the brass parts in grooves filled with picein. The main joint was water cooled as shown.

The brass part was designed to go between the jaws of an electromagnet, and hence was made as thin as convenience would permit. In its construction, a brass plate 3/8 inch thick and 8 inches square was mounted on the face plate of a lathe, and a channel of semicircular cross section cut in its

face. The plate was then removed from the lathe, cut into two halves, and the halves folded over like a book. When these were soldered face to face a semicircular tube of 4 cm. radius of curvature was formed in a block 3/4 inch thick. There was, moreover, ample solid stock into which the several

FIG. 2.



Final apparatus.

necessary fittings could be threaded and soldered as shown.

The canal out of which the ions issued is shown at *C*. It was drilled through an aluminum piece fitted to a brass cup. This aluminum part could easily be changed for another similar one carrying a canal of different length or diameter. Lengths of 1 and 2 cm. and diameters of 2 and 1 mm. were used. The brass cup was tightly seated onto a glass grind

and this was supported by an inseal to the outside wall of the tube as shown. By virtue of this design, the canal was the only gas passage from the upper part of the tube to the lower. Hydrogen was admitted as shown, and diffusion pumps were connected to the two outlets shown in the figure. F was a tungsten filament and A a nickel plate, both supported by the ground joint at the top.

In the operation of the tube the brass portion was grounded and the electrodes C, F and A formed a system connected to the high side of a kenotron rectifying outfit capable of supplying 75 kv. F and C were at the same potential, but A was 90 volts positive with respect to these. Under this difference of potential, an electron stream of 1–10 m.a. produced by impact an abundant supply of hydrogen ions. These passed downward, some of them going through the canal and being accelerated toward the opening G. Beyond G they passed into the field of an electromagnet and were curved to the right in the figure. By adjusting the magnetic field, it was possible to separate H^+ ions or protons from the H_2^+ and H_3^+ ions which were also present, and cause them to strike the target T. Radiation from T passed through the cellophane window W to a detecting device.

The principal reason for bending the ions around in a magnetic field was to remove the target from the path of radiation proceeding through the canal from the discharge. The additional advantage was gained, however, that the protons could be separated out from the other types of ion and from neutral canal rays. The ion beam struck no solid material until well down in the brass channel. Thus any secondary electrons were released in a space where the electric field required to accelerate the ions was very greatly reduced, much as in a Faraday cylinder. Furthermore, such secondary electrons were bent back by the field of the magnet, strong in this region. The space above G was magnetically shielded by a steel housing outside the tube. The region around the target T was free from any electric field. Thus the secondary electrons produced by the impact of the protons were not accelerated and produced no radiation. The target and the detecting device, owing to their remote position could readily be shielded from stray x-rays arising in other parts of the tube.

The filament of the tube was heated by a transformer with an insulated secondary such as is commonly used with kenotrons. The high voltage supply was equipped with a large reversing switch so that electrons from the filament could be drawn through the canal in place of the protons, and accelerated. By reversing and adjusting the magnetic field they could then be brought onto the target. The current reaching the target, whether positive or negative, was measured by a Swann type quadrant electrometer connected through an arrangement of calibrated high resistance shunts by means of which a large range of currents could be measured.

For the detection of x-rays proceeding from the target, use was made of a Geiger point counter filled with argon and closed at the front by a window of cellophane. Except for the use of argon, the technique followed was that of Bennett.[10] The counter chamber was dust proof, and the argon which flowed through was dried with sulphuric acid. Under these conditions stable satisfactory operation for at least a month was achieved without changing the platinum ball point. The discharges of the counter were observed by means of a single fibre electrometer.

The target under investigation was of copper, there being several reasons for the choice of this element. In the first place, the amount of kinetic energy which could be given to the protons was limited by the magnetic field needed to bend their path to a 4 cm. radius of curvature. The broad pole faces of the electromagnet had to be separated 1.9 cm. to allow the brass analyzing box to fit in between. With so large a gap, a field of not more than about 6,000 gauss could be set up for as long as needed without overheating the magnet. With this field, the prescribed radius of curvature was obtained when the protons had fallen through about 27 kv. With electrons, this voltage is ample abundantly to excite the K radiation of copper whose critical potential is 8.86 kv. With protons, of course, information was lacking but it was thought desirable, in any case, to choose a target material having a critical potential well below the voltage that could be supplied to the striking particles. On the other hand, an element of still lower critical potential would emit radiation of wave-length

[10] R. D. Bennett, *J. Opt. Soc. Am.*, *16*, 339 (1928).

too long to pass through the necessary vacuum tight window of the apparatus and then into the Geiger counter. Finally, it was desirable for the radiation to be absorbed strongly by the argon in the counter. In these respects, the wave-length region of the copper K radiation (λ 1.38–1.54 $\AA$) is ideal. Cellophane windows of the type used absorb only 2 or 3 per cent. of radiation λ 1.5 $\AA$., and yet some 15 per cent. of this radiation is absorbed by 1 cm. of argon at atmospheric pressure.

The cellophane used for the windows was .001 inch thick. This material has amazing strength. The window of the tube was ½ inch in diameter. It was formed by sticking, with picein cement, a piece of the cellophane onto the brass surface carrying the hole. The edges of the hole were rounded to avoid cutting the cellophane. No other precautions were taken. Although the cellophane stretched into a taut concave surface, it did not break and the window was vacuum tight. It was still in service after six months, although several times it had been snapped into a convex position by blowing incidental to repairs in the glass system.

EXPERIMENTAL METHOD.

In principle, each experiment consisted of (*a*) an overall calibration of the apparatus with electrons of definite energy striking the target, followed by (*b*) an attempt to detect radiation with protons of the same, or another, definite energy striking the target.

The procedure of calibration (*a*) was as follows: With no hydrogen entering the system, and with fields directed for electrons to strike the target, the desired voltage was applied and the filament heated. The magnetic field was then adjusted until the incidence on the target of the electron beam was evidenced by negative charging of the electrometer. The electron current striking the target was measured. Then the argon filled counter was put into operation by applying a suitable voltage and the number of counts per minute recorded. The resulting number constituted a measure of the intensity of x-radiation emitted at the target corresponding to the known electron current. Tests showed the number of counts to be roughly proportional to the current as was expected.

Proceeding to the proton test (*b*), the voltage on the counter was maintained the same as nearly as possible, and the voltage applied to space *CG* was reversed. It was kept the same in some experiments and raised in others.[11] The magnetic field was also reversed.

Hydrogen was admitted at constant rate, and the filament heated until the emission amounted to a convenient current (1 to 10 m.a.). The magnetic field was adjusted to bring the proton beam on the target. The proton current was measured and the number of discharges per minute of the counter observed.

Before presenting detailed data, it should be said that even with electron currents as small as 10^{-14} amp. striking the target, the radiations could be observed. This is an indication of the high sensitivity of the apparatus, requisite because of the limited magnitude of the proton currents which could be produced. The detection of the radiation from such unexpectedly small currents made desirable a proof that the phenomenon was correctly interpreted. Two experimental checks were made. First, comparisons were made of the Geiger counts (1) with the electron beam striking the target and (2) the beam falling off of the target, the alternate conditions being obtained by slightly changing the magnetic field. The parts struck in the latter case were not in fromt of the cellophane window. No other conditions were changed and it was found that the counter discharges stopped in passing from conditions 1 to 2. This proved that the phenomenon did arise from the small electron current striking the target, and was not caused by stray radiation from another source. Second, a rough measurement of the effective wave-length of the radiation by observing the absorption in thin aluminum screens interposed between the tube and counter windows, led to a figure in the x-ray region to be expected in the case of electrons of the given velocity striking cooper. This proved that the counter was responding, as supposed, to x-rays and not to ultra-violet light from gas atoms struck by the electrons or to any other radiations.

[11] It was found that higher voltages could be applied to space *CG* when protons were passing through than when the beam was composed of electrons. The limiting voltage in each case was that at which the resistance of the tube broke down and a discharge set in.

These experiments gave confidence in the correct operation of the apparatus as well as a knowledge of its sensitivity. Proceeding to experiments with proton beams, it was at once found that the radiation, if any, was so weak that it corresponded to a number of counts of the Geiger less than the number of strays in the latter, whereas, when working with electrons, the number of strays could be neglected in comparison with the counts due to the radiation. The problem thus became statistical in nature. That is, to measure the radiation, the average number of counts with the radiation off must be subtracted from the average number with the radiation on, other conditions being changed as little as possible. This was accomplished, as in the test with electrons, by altering the field of the magnet slightly so that the proton beam no longer struck the target. The counter was shielded from the stray field of the magnet, and also tests of its sensitivity with different imposed fields showed no interdependence.

RESULTS.

First Experiments.—Before comparing directly the effects produced when electron and proton beams struck the target, a series of runs was made to try to detect radiation when the proton beam was used. The procedure was to count for one minute with the proton beam on the target and one with it thrown off by a slight decrease in the magnetic field, this representing one comparison. The following table summarizes the results:

Energy of Protons (Kv.).	No. of Comparisons.	Total Counts.	
		Beam On.	Beam Off.
15	24	216	226
17.5	80	773	757

The counts in the last column must be due to the number of strays in the counter. This figure, for each voltage, should be subtracted from that in the third column, the resulting number of counts being caused by radiation. It will be seen that for 15 kv. the number so found is actually negative, while for 17.5 kv. it is positive but small, 16 compared with 773. It is

evident that column three is equal to column 4 within the limits of experimental accuracy. That is, there is no radiation or else it was too small to be detected in these preliminary experiments.

Comparison Experiment (*I*).—This experiment was performed under better conditions and with larger proton currents than the previous experiments.

Overall calibration:

Electron current. (Rate of deflection of electrometer.)	Energy of electrons (kv.).	Counts/min. Beam On.	Beam Off.
14.3	15	70	3

Comparison:

Proton current.	Energy of protons.	Total counts in 12 on and 12 off trials of 1 min. each. Beam On.	Beam Off.
67	15	144	140

Calculation:

Electrons: Neglecting 3 strays compared with, in all, 70 counts per minute, we find the current necessary to produce radiation corresponding to 1 count per min. to be $\frac{14.3}{70} = .20$ mm. sec.

Protons: Since the difference between on and off conditions was only 4 out of a total of 144, any radiation entering the counter must have been very weak. Allowing for statistical fluctuations, it is reasonable to estimate that it certainly corresponded to no more than 15 per cent. as many counts as the strays. That is, the radiations caused less than 21.6 total counts or 21.6/12 = 1.8 counts/min. From this, we conclude that to produce radiation equivalent to 1 count/min. would require a proton current of at least 67/1.8 = 37 mm./sec.

Therefore, for equal radiation, the ratio of required currents is $> 37/.20 = 185$. Now, a correction must be introduced because the counter was more sensitive during the run with protons, an indication of this fact being the greater number of strays observed. Taking the number of strays as a measure of counter sensitivity, we have the result that for the protons the sensitivity was 12 counts/min. ÷ 3 counts/min. = 4 times as sensitive as for electrons.

We thus have, finally, that the ratio of the efficiency with which electrons produce x-radiation to the efficiency with which protons produce the same radiation is

$$\rho \geqq 4 \times 185 = 740 \text{ at } 15 \text{ kv.}$$

Comparison Experiment (II): After several changes in pumping arrangements, canal and electrode dimensions, etc., a proton beam of a larger order of magnitude was obtained. As a result, it was possible to prove that ρ actually has a very much larger value indeed than the first minimum estimate of 740. The data and calculations follow:

Calibration:

Current (mm./sec.).	Energy (kv.).	Counts/min.	Current for 1 Count (mm./sec.).
.44	15	20	.022
1.87	15	39	.048
Average			.035

Comparison:

Current (Average for 8 Comparisons) (mm./sec.).	Energy (kv.).	Total Counts in 8 on and 8 off Comparisons of 2 min. each. Beam On.	Beam Off.
4,620	25	236	244

Electrons: Energies of slightly greater than 15 kv. applied when the beam consisted of electrons resulted in the breakdown of the tube resistance across space *CG* in Fig. 2. For comparison with the results using protons of 25 kv. energy, it is necessary to take the result for 15 kv. electrons and correct it so that it represents results for 25 kv. electrons. This may be accomplished by virtue of the well tested relation that the energy of x-radiation is proportional to the square of the voltage applied to the electrons.[12] Here then, for 25 kv. electrons, the current required for 1 count/min. becomes $.035 \times \frac{(15)^2}{(25)^2} = .013$ mm./sec. for 1 count/min. at 25 kv.

Protons: As before, we may estimate that the radiation was certainly less than that which would cause a number of counts equal to a certain percentage of the number of strays.

[12] M. Siegbahn, "The Spectroscopy of X-Rays," p. 213.

Since a larger total number of observations is available here, we may assume 10 per cent. instead of 15 per cent. allowance for statistical fluctuations. That is, the radiation caused less than 23.6 total counts or 23.6/16 = 1.5 counts/min. Therefore, radiation equivalent to one count/min. would require a proton current of at least 4,620/1.5 = 3,080 mm./sec. From which, as before

$$\rho \geqq \frac{3080}{.013} = 237{,}000 \text{ (at 25 kv.)}.$$

This enormous ratio between the efficiency of 25 kv. electrons and protons in exciting x-rays would be increased somewhat by a correction for relative sensitivity of the Geiger counter. Exact data concerning strays was not taken, however, and, indeed, this method of estimating sensitivity is open to objection. For this reason, it is best to let the above result stand.

From an estimate of the capacity of the electrometer system, it was calculated that the electron currents in this experiment were of the order of 10^{-14} amp., and the proton currents correspondingly of the order of 10^{-9} amp.

Comparison Experiment (III): The work was interrupted at this point by removal to a new laboratory. At the same time, the tube was reconstructed in the hope of obtaining even larger proton currents striking the target. This hope has not been realized, but before the work had to be discontinued for some months on account of another removal, a third comparison experiment was performed with energies of 17.5 kv

Calibration:

By direct observation of on and off conditions, it was determined that the radiation due to an electron beam of 1.43 mm./sec. could just be detected with certainty.

Comparison:

No radiation could be detected due to a proton beam of 9,200 mm./sec.

Calculation:

These two observations were made within one minute of each other. It was possible to assume that the sensitivity of

the counter was the same. Indeed, a check of its constancy was made before and after the comparison. We have as a result:

$$\rho \geqq \frac{9200}{1.43} = 6400 \text{ (17.5 kv.)}.$$

DISCUSSION.

The experiments here described presented difficulties in performance, but there would seem to be small chance of error in their interpretation. Expressed briefly, two currents were measured: (1) the current of electrons which certainly produced x-radiation, and (2) the current of protons which certainly produced less or no radiation. The simple ratio of these currents represented a minimum value of ρ, the ratio of the respective electron and proton efficiencies in producing x-rays. It has been pointed out that the comparison was very direct, the target, windows and detecting device being exactly the same, and likewise their geometrical arrangement.

Exception may be taken to the corrections for different Geiger sensitivity and for difference, in Experiment II, of the electron and proton energies. As a matter of fact, direct experimental test of the validity of such corrections was made. That is: (1) when the counter detected more strays, it also gave more counts in a steady beam of x-rays, showing that the number of strays is at least a rough measure of its sensitivity. Also, (2) a rise in the voltage applied to the electron beam did result in an increase in the radiation as detected by the counter. In short, there appears to be no basis for hesitation in offering the large result of Experiment II as a conclusion. Nevertheless, it is of interest to point out that, without any corrections, the simple ratio of the measured currents in Experiment II was 88,000 (= 3,080/.035).

No great claim for accuracy can be made, because fluctuating arc conditions in the tube caused fluctuations in the proton beam intensities. The error from this source could hardly affect the order of magnitude, however, and the latter has considerable theoretical significance.

For a discussion with reference to various theories of impact, the reader is referred to the excellent summary given by Bothe and Franz.[6] Briefly the expectations of several points of view are the following:

If the problem is to be treated classically, the radiation being due to the retardation of the proton in the field of the heavy atom nucleus, the radiation would be displaced to much longer wave-lengths than is the case for electron impacts, and the total radiated energy would vary inversely as the 3/2 power of the mass ratio.[13] This, applied to the present experiments in which a limited band of wave-lengths is studied, would mean a value for $\rho \gg (M/m)^{3/2} = (1{,}846)^{3/2} = 79{,}310$. How much greater, depends on the exact assumption regarding the shape of the radiation pulse, and hence the wave-length distribution of the energy. The experiments described thus reach only the minimum order of magnitude which the theory would predict and do not, therefore, controvert the classical theory.

The remarks of the previous paragraph are restricted—as is the success of classical considerations—to the continuous spectrum. The characteristic radiation, we may suppose from the Bothe and Franz experiments, is what we are more apt to be concerned with. This radiation follows upon an ionization in (e.g.) the K shell. Two points of view are summarized in this respect by the writers mentioned. First, Thomson[14] suggested that an electron (e.g. in the K shell) must receive the energy needed to remove it from the atom by the simple transference of momentum at a classical close encounter. This view would require protons of much greater energy than electrons (actually twice the *velocity* or more) for the production of ionization; in the case of a head on collision, the required amount would reach the minimum of 462 W, W being the critical excitation potential for electron impacts. There would have been no radiation either in the present experiments or in those of Bothe and Franz, the energy transferable on Thomson's view being in both cases too small. The writer's negative result is not in contradiction to this theory but the result of Bothe and Franz obviously is.

It would seem that only in the wave mechanics[2] lies hope of attacking the problem theoretically. As Bothe and Franz point out in their discussion, the only clear conclusion which the wave theory of impacts has reached is that the critical

[13] A. H. Compton, "X-Rays and Electrons," Chap. II.

[14] J. J. Thomson, *Phil. Mag.*, *23*, 449 (1912).

excitation *energy* should always be the same regardless of the striking particle. Thus the present experiments should have produced radiation. However, the theory leaves in a very unsatisfactory form the question of efficiency of impact. It is probable that the result of this experiment will serve as a check of the wave theory of impacts. At present, it appears that we must wait for a further development of the theory before such a check can be made.

6th International Congress of Mining, Metallurgy and Applied Geology will be held in Liege, Belgium, on June 22 to 28, 1930, during the continuance of the International Exposition of 1930. The Congress is divided into three sections: A, mines, B, metallurgy and C, applied geology.

A pamphlet containing a list of the Honorary Committee, the Patrons and the General Committee on Organization as well as the rules governing the meetings of the congress and a tentative list of the subjects to be discussed has just been issued.

Excursions and visits have been planned for the members which will include inspections of iron, manganese and other mines, metallurgical laboratories, industrial plants and a visit to Spa. Communications for the Congress may be sent to 16 Quai des Etats-Unis, Liege, Belgium.

Cutting Capabilities of Lathe Tools. By DEMPSTER SMITH. (North-East Coast Institution of Engineers and Shipbuilders, Newcastle-upon-Tyne. Read at a General Meeting held 6th December, 1929.) A brief outline is given of the development of cutting metals, their chemical composition and the functions of the various elements. Reference is made to cutting-tool alloys other than steel, such as Stellite, Cooperite, Widia and Perdurum. The heat treatment of tool steels and some of their properties when hot and cold and also the induced hardness of the work is discussed with the object of showing the difficulty experienced in obtaining a rational law between cuts of different dimensions and cutting speed when operating on a given material.

An outline is given of the work done by the Manchester Lathe Tool Committee on the effect of the depth of cut and traverse, shape of tool and speed on the durability of ordinary high-speed-steel tools; also on the forces exerted on the tool when cutting iron and steel. As cutting steels are being continually improved, the cutting speeds advanced will not be equal to the most up-to-date practice but they will form a basis for comparison and indicate the character of the results likely to be expected.

R.

DYNAMICAL ANALYSIS OF MACHINES.*

BY

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PART I—MACHINE COMBINATIONS WITH ONE DEGREE OF FREEDOM.

In the application of the generalized methods of dynamics to the analysis of machines, we are confronted with considerable mathematical complications, even for the most elementary machine combinations. For this reason it is desirable to use a form of equations developed from the generalized equations of dynamics, but in a form so that advantage may be taken of the highly developed graphical methods of velocity diagrams found in the kinematic analysis of mechanisms.

In the development of the kinematic theory of mechanisms the ordinary procedure has been in the classifications of machine combinations into various pairings and linkage combinations. Thus we have sliding and turning pairs, the four bar linkage, sliding block linkage, etc. A machine is ordinarily analyzed as an aggregation of these pairings and elementary linkage combinations. The simple velocity diagrams for these elementary pairings and linkages may then be readily extended to the more complicated machine combinations.

While this method cannot be underestimated, a procedure consistent with a dynamical analysis should include the classification of a mechanism and the analysis of its motion as a function of one or more coördinates.

An ordinary mechanism has but one degree of freedom and therefore one generalized coördinate. In the more complicated machine combinations we have systems with two or more degrees of freedom. In this paper, however, we will only consider the ordinary mechanisms with one degree of

* Extension of a portion of a dissertation for the degree of doctor of philosophy submitted to Clark University, 1928.

freedom. Such mechanisms include combinations from the elementary pairings and linkages already mentioned.

The dynamical procedure involves the introduction of velocity ratios and their derivatives. The velocity ratios are obtained from the kinematic analysis of the velocity diagrams, either mathematically or graphically. The derivatives of the velocity ratios with respect to the fundamental coördinate introduces further velocity ratios, so that a complete solution is possible. By this method the dynamical analysis can be carried on mathematically, semi-mathematically or entirely graphically.

The function of a machine is to transmit energy with a prescribed motion. Therefore, in the analysis of a machine we are concerned with an analysis of its motion and the proportioning of its parts consistent with the various loadings brought on the members. That is, aside from the motion we are vitally concerned with a complete analysis of the internal reactions.

The internal reactions are due to the inertia loadings of the members themselves as well as the loads transmitted, so that the inertia properties of the members and the acceleration of the parts must be determined.

With given external forces applied to a machine; that is, the driving and resisting forces, the motion of the machine depend upon the inertia properties of the mechanism and its configuration. A general procedure of analysis would be:

1. To determine the motion of the machine as a whole as a function of a given parameter or coördinate when subjected to the given external loadings.

2. To estimate the accelerations of the various parts corresponding to the acceleration and velocity of the coördinate.

3. To calculate the inertia loadings of the members and

4. Finally, to analyze the nature of the internal reactions and stresses in the mechanism.

To calculate the velocities and accelerations of the parts we must know the kinematic conditions of the relative motions of the parts; that is, the constraint imposed upon the mechanism so that its motion is reduced to one degree of freedom.

By choosing a suitable parameter or coördinate the motion

as well as the configuration of the mechanism becomes a function of the "generalized" coördinate. A basic classification is, therefore:

1. Mechanisms which change their configurations as a function of the displacement coördinate, which may be termed variable types.

2. Mechanisms where the configuration is independent of the displacement coördinate, which we will term cyclic mechanisms.

In the former the motion depends upon the particular parameter defining the configuration and the velocity of the coördinate defining the motion; that is, the motion of any part is $v = K\dot{\theta}$, where $K = f(\theta)$.

In the latter the motion of the parts are independent of the displacement coördinate. That is, $v = K\dot{\theta}$, where now K = constant. Examples are found in simple gear trains, parallel crank rod drives, etc.

GENERALIZED EQUATION OF MOTION FOR ONE DEGREE OF FREEDOM.

The energy of a system can be expressed in terms of parameters or coördinates, which define the configuration and motion of the system. In elementary machine combinations we have seen that ordinarily the configuration of the mechanism is a function of a single coördinate. We may, therefore, express the energy in terms of inertia coefficients which contain velocity ratios with respect to the fundamental coördinate and the coördinate itself.

The configuration is expressed by:

$$x = f_1(\theta), \qquad y = f_2(\theta), \qquad \text{etc.}, \qquad S = F(\theta),$$

so that the motion of any point is given by:

$$\delta s = \alpha \delta\theta, \qquad v = \dot{s} = \alpha\dot{\theta},$$

or analytically:

$$\delta x = \frac{dx}{d\theta} \cdot \delta\theta, \qquad \dot{x} = \frac{dx}{d\theta}\dot{\theta},$$

$$\delta y = \frac{dy}{d\theta} \cdot \delta\theta, \qquad \dot{y} = \frac{dy}{d\theta}\dot{\theta}.$$

To obtain the inertia coefficient of the system, we have:

$$\Sigma \tfrac{1}{2} m v^2 = \tfrac{1}{2} A \dot{\theta}^2$$

so that:

$$A = \Sigma m \left(\frac{ds}{d\theta}\right)^2 = \Sigma m \left[\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right];$$

$$\therefore \quad A = \Sigma m \left(\frac{\dot{s}}{\dot{\theta}}\right)^2 = \Sigma m \left[\left(\frac{\dot{x}}{\dot{\theta}}\right)^2 + \left(\frac{\dot{y}}{\dot{\theta}}\right)^2\right],$$

or in a particular form for machine combinations:

$$A = \Sigma m \left(\frac{v}{\dot{\theta}}\right)^2 + \Sigma I \left(\frac{\omega}{\dot{\theta}}\right)^2,$$

where

m = mass of a member,
I = moment of inertia about its c. of g.,
v = velocity of c. of g.,
ω = angular velocity of member.

To calculate the reduced forces, we note:

$$\Phi d\theta = \Sigma P ds \cos \alpha,$$

where α is the angle between $\bar{P}$ and $\bar{ds}$,

$$\therefore \quad \Phi = \Sigma P \frac{ds \cos \alpha}{d\theta} = \Sigma P \cos (\bar{P}, \bar{v}) \frac{v}{\dot{\theta}},$$

or analytically:

$$\Phi = \Sigma \left(X \frac{dx}{d\theta} + Y \frac{dy}{d\theta}\right)$$

$$= \Sigma \left(X \frac{v_x}{\dot{\theta}} + \bar{Y} \frac{v_y}{\dot{\theta}}\right),$$

when X, Y are derivable from a potential, then:

$$\Phi = -\left(\frac{\partial V}{\partial x} \frac{dx}{d\theta} + \frac{\partial V}{\partial y} \frac{dy}{d\theta}\right) = -\frac{dV}{d\theta}.$$

Therefore, the general equation of energy is:

$$d(\tfrac{1}{2}A\dot{\theta}^2) = \Phi d\theta - \frac{dV}{d\theta} \cdot d\theta,$$

which enjoys the elimination of reactions, such as the tensions of inextensible strings or rods, the mutual reactions between frictionless joints, and of the various parts of a rigid body, the reactions of smooth fixed surfaces, etc., which on the whole do no work.

Since the generalized coefficient of inertia is a function of the coördinate itself, the expression on differentiating becomes:

$$A\ddot{\theta} + \tfrac{1}{2}\frac{dA}{d\theta}\dot{\theta}^2 = \Phi - \frac{dV}{d\theta},$$

which is a particular and useful form of generalized equation of dynamics with one degree of freedom, where it is to be particularly noted the generalized forces are derivable only from work forces, the reactions of the constraints, etc., being eliminated.

This expression is also readily obtained from the Lagrangian equation for one degree of freedom. Thus:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} = \Phi - \frac{\partial V}{\partial \theta},$$

where $T = \tfrac{1}{2}A\dot{\theta}^2$;

$$\therefore \quad \frac{\partial T}{\partial \dot{\theta}} = A\dot{\theta}, \qquad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = A\ddot{\theta} + \frac{dA}{d\theta}\dot{\theta}^2,$$

$$\frac{\partial T}{\partial \theta} = \frac{1}{2}\frac{dA}{d\theta}\dot{\theta}^2,$$

so that on substituting we obtain the expression already given.

In many cases in machine combinations the direct expression of the coefficient of generalized inertia in terms of the coördinate θ results in considerable complexity. It is, therefore, preferable to obtain an expression directly in terms of velocity ratios, so that immediate advantage may be taken

of the kinematic process used in velocity diagrams, etc. Since:

$$A = \Sigma m\left(\frac{ds}{d\theta}\right)^2 = \Sigma m\left(\frac{v}{\dot{\theta}}\right)^2 + \Sigma I\left(\frac{\omega}{\dot{\theta}}\right)^2,$$

the equation of motion may be written as:

$$\Sigma m\left(\frac{ds}{d\theta}\right)^2 \ddot{\theta} + \Sigma m\left(\frac{ds}{d\theta}\right)\left(\frac{d^2s}{d\theta^2}\right)\dot{\theta}^2 = \Phi - \frac{dV}{d\theta}$$

or analytically:

$$\Sigma m\left[\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right]\ddot{\theta} + \Sigma m\left[\frac{dx}{d\theta}\frac{d^2x}{d\theta^2} + \frac{dy}{d\theta}\frac{d^2y}{d\theta^2}\right]\dot{\theta}^2 = \Phi - \frac{d\bar{V}}{d\theta}$$

or in terms of velocity ratios:

$$\left[\Sigma m\left(\frac{v}{\dot{\theta}}\right)^2 + \Sigma I\left(\frac{\omega}{\dot{\theta}}\right)^2\right]\ddot{\theta} + \left[\Sigma m\frac{v}{\dot{\theta}}\frac{d}{d\theta}\left(\frac{v}{\dot{\theta}}\right) \times \Sigma I\left(\frac{\omega}{\dot{\theta}}\right)\frac{d}{d\theta}\left(\frac{\omega}{\dot{\theta}}\right)\right]\cdot\dot{\theta}^2 = \Phi - \frac{dV}{d\theta}.$$

If we define the linear and angular velocity ratios with respect to the coördinate θ by K_{v_n} and K_{ω_n} . . . for any member n, the equation of motion becomes:

$$\sum_1^n (m_n K_{v_n}^2 + I_n K_{\omega_n}^2)\ddot{\theta} + \sum_1^n\left(m_n K_{v_n}\frac{dK_{v_n}}{d\theta} + I_n K_{\omega_n}\frac{dK_{\omega_n}}{d\theta}\right)\dot{\theta}^2 = \Phi - \frac{d\bar{V}}{d\theta},$$

which is the general dynamical equation in terms of velocity ratios, a form particularly valuable in machine analysis.

The previous dynamic equation requires the kinematic data of velocity ratios and the derivative of the velocity ratios with respect to the fundamental coördinate. Another generalized form is the direct calculation of the kinetic reactions of the members and their reduction to the primary

coördinate. With this procedure we must calculate the actual accelerations. But from the acceleration diagram the acceleration of the members may be reduced to a function of the acceleration of the fundamental coördinate together with centrifugal terms that may be evaluated from the velocity ratios. Finally, the kinetic reactions and applied forces themselves are reduced to the fundamental coördinate by the velocity ratios. The kinematic data, therefore, requires acceleration diagrams combined with the velocity ratios.

From D'Alembert's equation:

$$\Sigma\left[\left(m\frac{d^2x}{dt^2}-X\right)\delta x+\left(m\frac{d^2y}{dt^2}-Y\right)\delta y\right]=0,$$

where X, Y are work forces, the reactions of constraints and internal forces of a rigid body, etc. being eliminated, since the otherwise arbitrary displacements δx, δy, etc., are subjected to the conditions of the constraints.

With one degree of freedom, we have:

$$\delta x=\frac{dx}{d\theta}\cdot\delta\theta \qquad \text{and} \qquad \delta y=\frac{dy}{d\theta}\cdot\delta\theta,$$

so that we have in terms of the parameter θ:

$$\Sigma\left(m\frac{d^2x}{dt^2}\frac{dx}{d\theta}+m\frac{d^2y}{dt^2}\frac{dy}{d\theta}\right)=\Sigma\left(X\frac{dx}{d\theta}+Y\frac{dy}{d\theta}\right),$$

where

$$\frac{dx}{d\theta}=\frac{\dot{x}}{\dot{\theta}}=K_{x\theta} \qquad \text{and} \qquad \frac{dy}{d\theta}=\frac{\dot{y}}{\dot{\theta}}=K_{y\theta}.$$

For a mechanism this equation reduces, in terms of the kinetic reaction of the members, to the form:

$$\Sigma\left(m\frac{d^2s}{dt^2}K_{s\theta}+I\frac{d^2\phi}{dt^2}K_{\phi\theta}\right)=\Sigma\left(FK_{s\theta}+LK_{\phi\theta}\right),$$

where s corresponds to any linear coördinate and ϕ to any angular coördinate, and $K_{s\theta}$ and $K_{\phi\theta}$ are the corresponding velocity ratios, and F and L are the applied forces and couples, and I is the moment of inertia of any member.

The particular advantage of this equation, as before, lies in the complete elimination of internal reactions and constraints, which on the whole do no work, while the kinematic procedure requires the calculation of acceleration diagrams together with the velocity ratios.

The accelerations may be expressed in terms of the velocity ratios, since:

$$\frac{ds}{dt} = K_{s\theta}\frac{d\theta}{dt}; \qquad \frac{d\phi}{dt} = K_{\phi\theta}\frac{d\theta}{dt},$$

so that

$$\frac{d^2s}{dt^2} = K_{s\theta}\frac{d^2\theta}{dt^2} + \frac{dK_{s\theta}}{d\theta}\left(\frac{d\theta}{dt}\right)^2,$$

$$\frac{d^2\phi}{dt^2} = K_{\phi\theta}\frac{d^2\theta}{dt^2} + \frac{dK_{\phi\theta}}{d\theta}\left(\frac{d\theta}{dt}\right)^2.$$

Substituting in the equation for the kinetic reactions of the members, we have:

$$(\Sigma mK_{s\theta}^2 + \Sigma IK_{\phi\theta}^2)\,\ddot{\theta} + \left(\Sigma mK_{s\theta}\frac{dK_{s\theta}}{d\theta} + \Sigma IK_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta}\right)\dot{\theta}^2 = \Sigma F_s K_{s\theta} + \Sigma L_\phi K_{\phi\theta}.$$

The derivative of any velocity ratio may be expressed in terms of the acceleration; that is,

$$\frac{dK_{\phi\theta}}{d\theta} = \frac{d}{d\theta}\left(\frac{\dot{\phi}}{\dot{\theta}}\right) = \frac{1}{\dot{\theta}}\frac{d\dot{\phi}}{d\theta} - \frac{\dot{\phi}}{\dot{\theta}^2}\frac{d\dot{\theta}}{d\theta},$$

but

$$\frac{d\dot{\phi}}{d\theta} = \frac{\ddot{\phi}}{\dot{\theta}}, \qquad \dot{\phi} = K_{\phi\theta}\dot{\theta} \qquad \text{and} \qquad \frac{d\dot{\theta}}{d\theta} = \frac{\ddot{\theta}}{\dot{\theta}},$$

so that

$$\frac{dK_{\phi\theta}}{d\theta} = \frac{\ddot{\phi} - K_{\phi\theta}\ddot{\theta}}{\dot{\theta}^2}.$$

Therefore

$$K_{\phi\theta}\ddot{\phi} = K^2{}_{\phi\theta}\ddot{\theta} + K_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta}\dot{\theta}^2.$$

This shows, as we should expect, the actual acceleration multiplied by the velocity ratio with respect to the funda-

mental coördinate; i.e., the reduced acceleration of the member, can be expressed in terms of the velocity ratios and their deriavtives, a form given in the previous equations.

An important feature to note is that a transformation of any actual acceleration of a member to a generalized coördinate introduces centrifugal terms when the configuration of the system changes. With a large class of machinery as gearing pairs, etc., there is no change in configuration as a function of the displacement, so that the centrifugal terms are absent. For such cyclic systems, the reduced acceleration of a wheel, etc., is:

$$K_{\phi\theta}\ddot{\phi} = K^2_{\phi\theta}\ddot{\theta}.$$

By an entirely graphical procedure; that is, by the use of velocity and acceleration diagrams, we may evaluate the equation of motion with respect to any generalized coördinate θ, in the form:

$$[\Sigma mK^2_{s\theta} + \Sigma IK^2_{\phi\theta}]\ddot{\theta} + \left[\Sigma mK_{s\theta}\left(\frac{\ddot{s} - K_{s\theta}\ddot{\theta}}{\dot{\theta}^2}\right) + \Sigma IK_{\phi\theta}\left(\frac{\ddot{\phi} - K_{\phi\theta}\ddot{\theta}}{\dot{\theta}^2}\right)\right]\dot{\theta}^2 = \Sigma(FK_{s\theta} + LK_{\phi\theta}).$$

The form of the equations for the relative motion of a member with respect to a rotating frame as for a block sliding in a groove in a rotating frame is of special interest. Let ξ and η be the coördinates of the moving block with respect to axis attached to the moving frame. If ϕ is the angular velocity of the frame, then the acceleration of the block is:

$$\ddot{\xi} - \eta\ddot{\phi} - \xi\dot{\phi}^2 - 2\dot{\eta}\dot{\phi} \qquad \text{along } \xi,$$

$$\ddot{\eta} + \xi\ddot{\phi} - \eta\dot{\phi}^2 + 2\dot{\xi}\dot{\phi} \qquad \text{along } \eta.$$

Now ξ, η are determined from the geometry of the system as some function of θ and the relative velocity ratios are:

$$K_{(\xi\theta)\text{ rel.}} = \frac{\dot{\xi}}{\dot{\theta}} = \frac{d\xi}{d\theta}, \qquad \text{and} \qquad K_{(\eta\theta)\text{ rel.}} = \frac{\dot{\eta}}{\dot{\theta}} = \frac{d\eta}{d\theta},$$

where it is to be noted that $d\xi/d\theta$ and $d\eta/d\theta$ represent the

rate of increase of the relative coördinates themselves with respect to the fundamental coördinate.

Further the variations of the coördinates of the block along fixed axis coinciding with ξ and η are:

$$\delta\xi = (\dot{\xi} - \eta\dot{\phi})\delta t, \qquad \delta\eta = (\dot{\eta} + \xi\dot{\phi})\delta t,$$

so that, the total rate of change of the variable coördinates (with respect to fixed axis) are:

$$\frac{u}{\dot{\theta}} = K_{\xi\theta} = K_{(\xi\theta)\text{ rel.}} - \eta K_{\phi\theta} = \frac{\delta\xi}{\delta\theta},$$

$$\frac{v}{\dot{\theta}} = K_{\eta\theta} = K_{(\eta\theta)\text{ rel.}} + \xi K_{\phi\theta} = \frac{\delta\eta}{\delta\phi}.$$

The relative accelerations are:

$$\ddot{\xi} = K_{(\xi\theta)\text{ rel.}}\ddot{\theta} + \frac{dK_{(\xi\theta)\text{ rel.}}}{d\theta}\dot{\theta}^2,$$

$$\ddot{\eta} = K_{(\eta\theta)\text{ rel.}}\ddot{\theta} + \frac{dK_{(\eta\theta)\text{ rel.}}}{d\theta}\dot{\theta}^2,$$

and

$$\ddot{\phi} = K_{\phi\theta}\ddot{\theta} + \frac{dK_{\phi\theta}}{d\theta}\dot{\theta}^2,$$

so that the total acceleration components along ξ and η axis in terms of the velocity ratios are:

$$(K_{(\xi\theta)\text{ rel.}} - \eta K_{\phi\theta})\ddot{\theta} + \left[\left(\frac{dK_{(\xi\theta)\text{ rel.}}}{d\theta} - \eta\frac{dK_{\phi\theta}}{d\theta}\right) - \xi K_{\phi\theta}^2 - 2K_{(\eta\theta)\text{ rel.}}K_{\phi\theta}\right]\dot{\theta}^2$$

$$(K_{(\eta\theta)\text{ rel.}} + \xi K_{\phi\theta})\ddot{\theta} + \left[\left(\frac{dK_{\eta\theta}}{d\theta} + \xi\frac{dK_{\phi\theta}}{d\theta}\right) - \eta K_{\phi\theta}^2 + 2K_{(\xi\theta)\text{ rel.}} \cdot K_{\phi\theta}\right]\dot{\theta}^2.$$

These components may also be written:

$$\dot{u} - v\dot{\phi} \qquad \text{and} \qquad \dot{v} + u\dot{\phi}$$

or,

$$K_{\xi\theta}\ddot{\theta} + \frac{dK_{\xi\theta}}{d\theta}\cdot\dot{\theta}^2 + K_{\eta\theta}K_{\phi\theta}\dot{\theta}^2,$$

$$K_{\eta\theta}\ddot{\theta} + \frac{dK_{\eta\theta}}{d\theta}\dot{\theta}^2 + K_{\xi\theta}K_{\phi\theta}\dot{\theta}^2,$$

or,

$$(K_{(\xi\theta)\text{ rel.}} - \eta K_{\phi\theta})\ddot{\theta} + \left[\frac{dK_{(\xi\theta)\text{ rel.}}}{d\theta} - \eta\frac{dK_{\phi\theta}}{d\theta} - K_{\phi\theta}K_{(\eta\theta)\text{ rel.}} - K_{\phi\theta}(K_{(\eta\theta)\text{ rel.}} + \xi K_{\phi\theta})\right]\dot{\theta}^2,$$

$$(K_{(\eta\theta)\text{ rel.}} + \xi K_{\phi\theta})\ddot{\theta} + \left[\frac{d\overline{K}_{(\eta\theta)\text{ rel.}}}{d\theta} + \xi\frac{dK_{\phi\theta}}{d\theta} + K_{\phi\theta}K_{(\xi\theta)\text{ rel.}} + K_{\phi\theta}(K_{(\xi\theta)\text{ rel.}} - \eta K_{\phi\theta})\right]\dot{\theta}^2.$$

The generalized equation of motion with members referred to a rotating frame is:

$$\Sigma m(K_{\xi\theta}^2 + K_{\eta\theta}^2)\ddot{\theta} + \Sigma m\left(K_{\xi\theta}\frac{dK_{\xi\theta}}{d\theta} + K_{\eta\theta}\frac{dK_{\eta\theta}}{d\theta}\right)\dot{\theta}^2 = \Sigma F(K_{\xi\theta} + K_{\eta\theta}),$$

where

$$K_{\xi\theta} = K_{(\xi\theta)\text{ rel.}} - \eta K_{\phi\theta}, \qquad K_{\eta\theta} = K_{(\eta\theta)\text{ rel.}} + \xi K_{\phi\theta},$$

$$\frac{dK_{\xi\theta}}{d\theta} = \frac{dK_{(\xi\theta)\text{ rel.}}}{d\theta} - \eta\frac{dK_{\phi\theta}}{d\theta} - K_{\phi\theta}K_{(\eta\theta)\text{ rel.}},$$

$$\frac{dK_{\eta\theta}}{d\theta} = \frac{dK_{(\eta\theta)\text{ rel.}}}{d\theta} + \xi\frac{dK_{\phi\theta}}{d\theta}K_{(\xi\theta)\text{ rel.}}$$

A considerable simplification is effected by choosing the moving axis along and perpendicular to the groove or slot for the sliding member, Fig. 1.

The kinetic energy of the block m sliding with the relative velocity y in the rotating frame with angular velocity ϕ, is:

$$T = \tfrac{1}{2}m[(\dot{y} + a\dot{\phi})^2 + y^2\dot{\phi}^2] = \tfrac{1}{2}m(\dot{y}^2 + 2a\dot{y}\dot{\phi} + a^2\dot{\phi}^2 + y^2\dot{\phi}^2)$$
$$= \tfrac{1}{2}m[K_{(y\theta)r}^2 + 2aK_{(y\theta)r}K_{\phi\theta} + a^2K_{\phi\theta}^2 + y^2K_{\phi\theta}^2]$$
$$= \tfrac{1}{2}m[K_{y\theta}^2 + y^2K_{\phi\theta}^2],$$

where

$$K_{(y\theta)r} = K_{(y\theta)\,\text{rel.}}, \qquad K_{y\theta} = \frac{\dot{y} + a\dot{\phi}}{\dot{\theta}} = K_{(y\theta)r} + aK_{\phi\theta}.$$

The generalized equation of motion is:

$$A\ddot{\theta} + \frac{1}{2}\frac{dA}{d\theta}\dot{\theta}^2 = \sum F_s K_{s\theta} + \sum L_\phi K_{\phi\theta},$$

where

$$A = \sum m\,(K_{y\theta}^2 + y^2 K_{\phi\theta}^2) = \sum m(K_{(y\theta)}^2 + 2aK_{(y\theta)r}K_{\phi\theta} + a^2K_{\phi\theta}^2 + y^2K_{\phi\theta}^2)$$

and

$$\frac{1}{2}\frac{dA}{d\theta} = \sum m\left(K_{y\theta}\frac{dK_{y\theta}}{d\theta} + y^2K_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta} + yK_{(y\theta)r}K_{\phi\theta}^2\right)$$

$$= \sum m\left[K_{(y\theta)r}\frac{dK_{(y\theta)r}}{d\theta} + a\left(K_{(y\theta)r}\frac{dK_{\phi\theta}}{d\theta} + K_{\phi\theta}\frac{dK_{(y\theta)r}}{d\theta}\right) + (a^2 + y^2)K_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta} + yK_{(y\theta)r}K_{\phi\theta}^2\right].$$

So far no consideration has been given to the internal friction. Since the work of any frictional force depends upon the relative displacement between any two parts, the effect of friction may be included in the generalized equations by adding the friction forces multiplied by their relative velocity ratios with respect to the primary coördinate. The friction forces themselves are usual indeterminate, because they depend upon the magnitude of the internal reactions. Thus friction at a sliding pair depends upon the normal reaction, while the friction at a joint is proportional to the reaction at the joint. These reactions depend not only on the loads transmitted, but the inertia loadings of the members themselves. The accurate calculation of the friction forces, therefore, depend upon the determination of the internal reactions.

A good approximation, however, can be made by allowing for friction as a percentage of the applied forces, since ordinary mechanisms have fairly definite efficiencies. If F_R is any frictional force acting between any two parts of a mechanism,

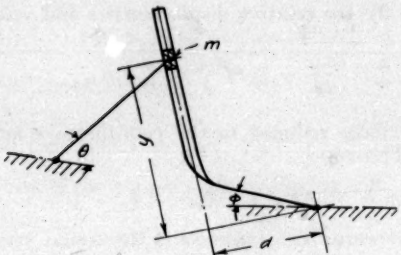


FIG. 1

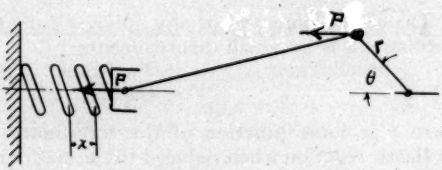


FIG. 2.

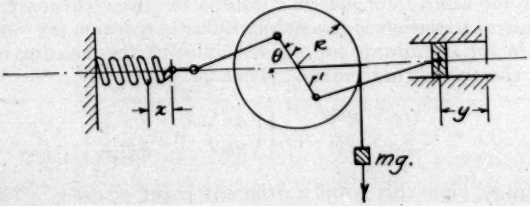


FIG. 3.

and $s_{rel.}$ and $v_{rel.}$ are the relative displacements and velocity, then:

$$\Phi_r = \sum F_r \frac{ds_{rel.}}{d\theta} = \sum F_r \frac{v_{rel.}}{\dot{\theta}} = \sum F_r \overline{K}_{s_{rel.}\theta}$$

and the applied force reduced to the coördinate θ and including friction becomes:

$$\Phi = \sum (FK_{s\theta} + LK_{\phi\theta}) - \Phi_r.$$

ON THE ELASTIC POTENTIAL AND REACTIONS IN MECHANICAL SYSTEMS.

The equation of motion of a mechanical system with one degree of freedom can be written:

$$A\ddot{\theta} + \frac{1}{2}\frac{dA}{d\theta} \cdot \dot{\theta}^2 = \Phi - \frac{d\overline{V}}{d\theta}.$$

The elastic potential of any spring element acting on the coördinate x is for small displacements:

$$\overline{V} = \tfrac{1}{2}cx^2,$$

where x is some function of the coördinate θ. Therefore, the elastic reaction when reduced to the coördinate θ becomes:

$$\Phi_{x\theta} = -\frac{d\overline{V}}{d\theta} = -cx\frac{dx}{d\theta} = -cK_{x\theta} \cdot x,$$

where

$$x = \int_{\theta_i}^{\theta} K_{x\theta} \cdot d\theta, \qquad \theta_i = \text{initial value when } x = 0.$$

Now the equivalent elastic constant for the coördinate θ is the rate of variation of the reduced elastic reaction per unit change in the coördinate for a given value of the coördinate, usually the equilibrium value. That is,

$$c_e = -\frac{d\Phi}{d\theta} = \frac{d^2\overline{V}}{d\theta^2} = c\left[\left(\frac{dx}{d\theta}\right)^2 + x\frac{d^2x}{d\theta^2}\right].$$

We may view this from a different point of view. The elastic reaction reduced to the coördinate θ, is (Fig. 2):

$$\Phi_e = -cx\frac{dx}{d\theta} = -cK_{x\theta}\cdot x$$

and the elastic constant is

$$c_e = -\frac{d\Phi_e}{d\theta} = c\left(K_{x\theta}^2 + x\frac{dK_{x\theta}}{d\theta}\right).$$

Thus in the simple system (Fig. 2),

$$x = r(1-\cos\theta), \frac{dx}{d\theta} = K_{x\theta} = r\sin\theta,$$

$$\frac{d^2x}{d\theta} = \frac{dK_{x\theta}}{d\theta} = r\cos\theta,$$

so that

$$-\frac{d\Phi}{d\theta} = cr^2(1-\cos\theta)\cos\theta + cr^2\sin^2\theta;$$

$$\therefore\quad c_e = cr^2(\cos\theta - \cos 2\theta).$$

We may verify this by direct analysis, as follows:

$$\Phi_e = Pr\sin\theta \quad \text{(approx.)};$$

$$\therefore\quad \Phi_e = cxr\sin\theta = cr^2(1-\cos\theta)\sin\theta,$$

so that

$$c_e = -\frac{d\Phi_e}{d\theta} = cr^2(\cos\theta - \cos 2\theta).$$

By a direct calculation from the potential, we have:

$$\Phi_e = -\frac{dV}{d\theta}, \qquad \text{where} \qquad V = \tfrac{1}{2}cx^2 = \tfrac{1}{2}cr^2(1-\cos\theta)^2;$$

$$\therefore\quad \Phi_e = -\frac{dV}{d\theta} = -cr^2(1-\cos\theta)\sin\theta,$$

so that

$$c_e = \frac{d^2V}{d\theta^2} = cr^2(\cos\theta - \cos 2\theta).$$

Thus in the vibration of a mechanical system with one degree of freedom about the equilibrium position θ_0 of the generalized coördinate, the equivalent or reduced elastic constant, is

$$c_e = \frac{d^2V}{d\theta_0^2} = \sum c\left(K_{x\theta}^2 + x_0 \frac{dK_\theta}{d\theta}\right)$$

where

$$x_0 = \int_{\theta_i}^{\theta_0} K_{x\theta} d\theta, \qquad \text{and} \qquad K_{x\theta} = \frac{\dot{x}}{\dot{\theta}} = \frac{dx}{d\theta}.$$

When the velocity ratio is invariable as in cyclic systems; that is, gear pairs, etc., the elastic constant becomes:

$$c_e = \sum c K_{x\theta}^2.$$

Neglecting the centripetal term in the generalized equation of motion, the oscillation about the equilibrium position is given by the expression:

$$A \frac{d^2(\Delta\theta)}{dt^2} + c_e \Delta\theta = E\Phi_0 \sin wt \cdots,$$

where

$$\Delta\theta = \theta - \theta_0.$$

(To be continued)

CORRELATIONS BETWEEN PHOTOGRAPHIC CHARACTERISTICS IN THE NORMAL AND IN THE SOLARIZED REGIONS OF EXPOSURES.

BY

A. P. H. TRIVELLI AND E. C. JENSEN.*

INTRODUCTION.

THE photographic literature contains many statements, of a qualitative nature, which express the correlations between photographic characteristics in the normal exposure region of photographic plates and films. It is, for instance, well known that it has been impossible to produce a high-speed photographic emulsion like the Eastman Speedway plate with the grain size of an Eastman Process plate. So far as commercial products are concerned, the correlation between speed and grain size of photographic emulsions has been expressed by the statement that usually higher-speed emulsions have larger grains than lower-speed emulsions. In the laboratory it is easy to produce a photographic emulsion with larger grains and lower speed than another photographic emulsion having smaller grains and higher speed. Usually in commercial practice there is a tendency to increase the speed to its limit. The speed can be brought to higher values in larger grains than in smaller grains resulting in the fact that on the whole, in photographic emulsions for commercial purposes, higher-speed plates contain larger grains than lower-speed plates.

Similar correlations have been expressed between fog and speed, between fog and grain size, *etc.* The normal exposure region has been the object of so thorough a quantitative investigation that many data can be collected to prove quantitatively how far these statements are justified.

This is quite different from the overexposure region. No correlations of the same kind have been stated for the region of the maximum density and the first reversal of the solarization, with probably the one exception that photographic

* Communication No. 406 from the Kodak Research Laboratories.

emulsions with smaller silver halide grains give higher maximum densities than photographic emulsions with larger silver halide grains. This may be due to the fact that the photographic characteristics in that exposure region are not expressed quantitatively as they are in the normal exposure regions; or it may be due to the absence of correlations in the solarization region. The only fundamental quantitative data, which have been used in the solarization exposure region, are the density and the threshold value.

J. M. Eder [1] was the first to point out that the threshold value of the first reversal of the solarization varies with the photographic material. H. Tappen [2] investigated this statement quantitatively, and found a variation of 15,000 to 247,000 m.c.s. for seven different commercial plates. The plotting of the exposures on an m.c.s. scale is especially inconvenient in the solarization region. Compared with the normal exposure region, the photographic material is extremely insensitive in the solarization exposure region, a fact mentioned by F. Weigert [3] in connection with the testing of Einstein's equivalent law for these exposures. Plotted on a logarithmic scale the threshold values of the first reversal of the solarization are not so great; the range extends from 4.18 to 5.39.

H. Tappen and B. Walter [3a] state that there is no correlation between the sensitivity of a photographic plate for the normal exposure region and the sensitivity in the first reversal of the solarization; and that the most sensitive plate is not always the first to solarize. H. Tappen concluded that a factor besides sensitivity determines the change of the threshold value of the first reversal of the solarization, and he suggested the hypothesis that this change depends on the silver iodide content of the photographic emulsion. This suggestion was based on the observation that a deep-yellow silver halide, which contains probably more iodide than the pale yellow silver bromide gave a lower threshold value of the first reversal of the solarization. This is in agreement with V. Schumann's

[1] *Phot. Korr.*, 645 (1902).
[2] *Phot. Korr.*, 365 (1908).
[3] *Z. physik. Chem.*, 99: 500 (1921).
[3a] *Ann. Physik*, IV, 27: 83 (1908).

investigations,[4] who found that the addition of silver iodide to silver cyanide emulsions accelerates solarization. This work of H. Tappen seems to be all that has been published on correlations in the first reversal of the solarization. The purpose of this investigation is to test how far these correlations hold for commercial photographic plates and films and to extend the search for other correlations over the entire range of exposures to the first reversal of the solarization. We assume, thereby, that H. Tappen expressed his ideas on sensitivity by means of the threshold values, the custom of his contemporaries in the Teutonic countries.

PREVIOUS DETERMINATIONS OF THE THRESHOLD VALUES OF THE FIRST REVERSAL OF SOLARIZATION.

It is obvious immediately in Tappen's data that he found, for photographic plates of quite different origins and, therefore, in all probability made in quite different ways, sometimes the same threshold value of the first reversal of the solarization. For example, out of seven various investigated commercial photographic plates he found the threshold values for

A German plate at 15,000 m.c.s.
An English " " 15,000 m.c.s.
An Austrian " " 62,000 m.c.s.
A French " " 62,000 m.c.s.

Lüppo-Cramer[5] made similar experiments; from curves on a logarithmic exposure scale around the maximum density ($D_{max.}$) he obtained similar results in a still more pronounced way. This investigation shows differences in the threshold value of the first reversal of the solarization from Log $E = 2$ to 3. The sixteen investigated photographic emulsions of different quality and of different origin can be divided into three groups each with the same threshold value of the first reversal of the solarization:

5 photographic emulsions at log $E = 2.0$
9 " " " " " " $= 2.5$
2 " " " " " " $= 3.0$

For convenient comparison with these results, the solarization curves in the graphs are arranged in the same way.

[4] *Eder's Jahrb.*, 391 (1898).
[5] *Z. wiss. Phot.*, 25: 224 (1928).

THE SENSITOMETER.

Both H. Tappen and Lüppo-Cramer made their exposures with an intensity scale at a fixed time of exposure. It is easier to obtain greater accuracy with a time scale, if the range of exposures is as great as is necessary for the investigation of the solarization. We used, therefore, a time scale sensitometer nonintermittent in 26 step $\sqrt{2}$ ratio and running from 1 to 5,791 seconds' exposure. The light source was a 500 c.-p. standard monoplane tungsten filament lamp operated at 87 volts and 4.01 amperes with a color temperature of 2900° K. The light intensity was kept constant by a voltage regulator which kept the voltage constant to within one volt, corresponding to a maximum deviation of 6 per cent. of the light intensity of the actinic part of the spectrum of the light source. Two series of exposures were made by placing two different diaphragms between the light source and the photographic plate or film. The photographic plate or film was placed at a distance of 99.1 cm. from the diaphragm, which contained a diffusion screen of opal glass or a piece of white glass, etched on both sides. For the normal exposures, a diaphragm giving 0.0331 m.c.s. light intensity on the photographic plate was used; and for the solarization exposures, a diaphragm giving 430 m.c.s. light intensity on the photographic plate was used.

PHOTOGRAPHIC MATERIAL AND DEVELOPMENT.

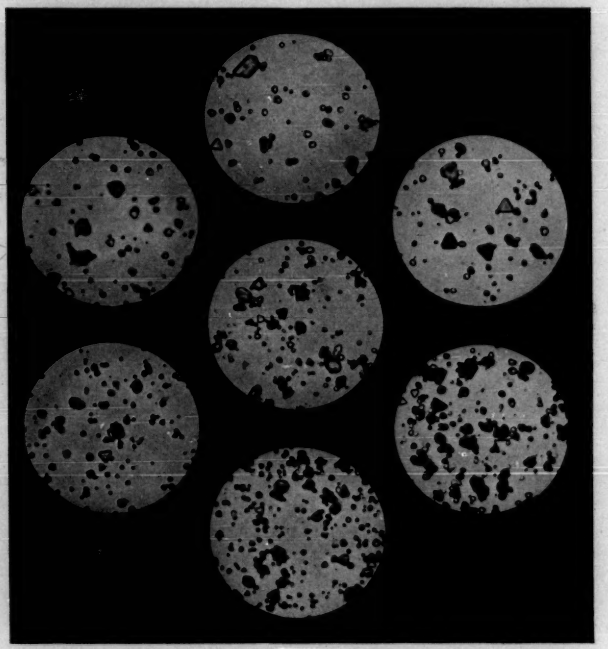
The fourteen different commercial plates and films investigated were:

5 high-speed	photographic	emulsions	indicated	as *A,B,C,D,E*
3 high-medium-speed	"	"	"	" *F,G,H*
4 low medium-speed	"	"	"	" *I,J,K,L*
2 low speed	"	"	"	" *M,N*

The alphabetic order is so chosen that *A* represents the photographic emulsion with the highest H. and D. speed and *N* represents the photographic emulsion with the lowest H. and D. speed. The H. and D. speeds were determined for four-minute development at 17° C. with a hydroquinone developer, the formula for which is given below. The exposures were made without a daylight filter. The values

obtained for the H. and D. speed, therefore, are not directly comparable with the speed determined for daylight-corrected exposures and standardized development. In this investigation development was chosen to give favorable results in

FIG. 1.



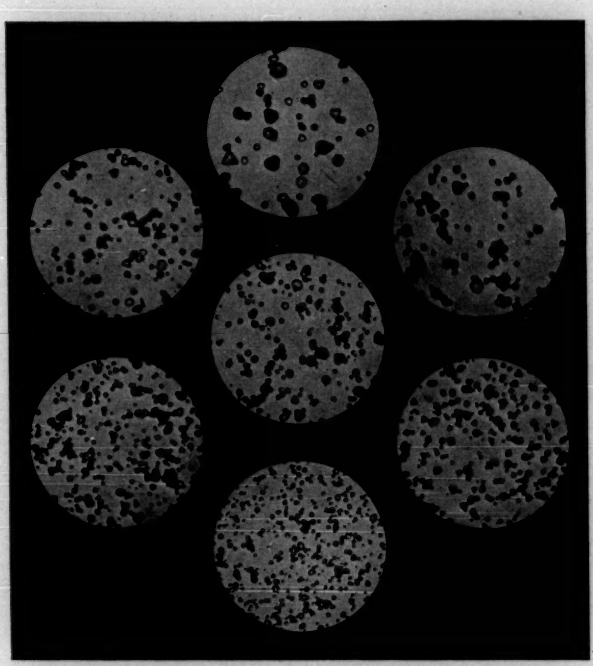
Grains of photographic emulsions, 2500 X.

the solarization region, and comparisons with the data obtained from the normal exposed part of the H. and D. curve were possible only if the same development conditions were applied to both regions of exposures. The fog and speed data

obtained are given in the second and in the third column of Table I, which shows the range of sensitivities used in this investigation.

Figures 1 and 2 show photomicrographs of the silver halide

FIG. 2.



Grains of photographic emulsions, 2500 ×.

grains of the investigated photographic emulsions made at 2500× enlargement. In all the experiments the development was effected for 4 minutes at 17° C., controlled by a thermostat, with brush according to W. Clark's method [6] in the following developer:

[6] *Phot. J.*, **65**: 76 (1925).

Hydroquinone	5.5 grams
Sodium sulfite, anhydrous	50.0 "
Sodium carbonate	50.0 "
Potassium bromide	0.5 "
Water to make	1.0 liter

The sulfur dioxide content of the sodium sulfite used was determined analytically and the actual amount calculated for an equivalent of 100 per cent. pure salt. Development was stopped at the required time with diluted acetic acid (1 per cent.). The plates and films were fixed in an acid fixing bath. Densities were read on a Martens densitometer. The readings were checked with a Jones [7] densitometer. The photographic emulsion *N* was read only on the Jones densitometer on account of its high densities.

TREATMENT OF DATA.

Every exposure was repeated 5 times and the data obtained were averaged and plotted in groups with the same threshold value for the first reversal of the solarization. In the most important part of the curves smoothing was avoided. The H. and D. speed, the threshold value, the gamma, the maximum density, and the exposure in m.c.s. at which the maximum density is reached were determined. The maximum density is given in Table II. From the photomicrographs the mean projective area of the silver halide grains expressed in μ^2 was determined, and from this was calculated the average grain size. With the exception of the maximum density, all these data are collected in separate columns in Table I. The number of grains measured are also given in a separate column, without using these data further for correlation determinations. The iodide content of the photographic emulsions was determined by Mr. A. Ballard of the department of analytical chemistry of these Research Laboratories, according to a method worked out by W. Clark.[8]

To compare the values for the H. and D. speed and gamma of the solarization region with those for the H. and D. speed and gamma of the normal exposure region, the values should be determined in a similar way.

[7] *J. Opt. Soc. Amer.*, 7: 230 (1923).
[8] *J. Chem. Soc.*, 749 (1926).

TABLE I.

Phot. Emuls.	Fog.	H.D. Speed.	H.D. sol. Speed $\times 10^6$.	γ	$\bar{\gamma}$ Isol.	t.v $\times 10^2$ in mcs.	t.vI sol in mcs.	(D/E) max.	M.C.S. at (D/E) max.	% Ag I	Average Grain Size in μ^2	Number of Grains Measured.	M.C.S. at D max.
A	0.12	272	288	0.80	0.78	52	10,834	1.36	16.81	5.3	0.59	95	10,834
B	0.11	272	288	0.83	0.95	52	21,715	1.56	15.67	6.7	0.58	124	21,715
C	0.11	254	330	0.92	0.90	52	21,715	1.56	15.67	6.7	0.48	130	21,715
D	0.08	254	998	0.88	0.74	52	5,417	1.84	11.11	4.8	0.35	281	3,883
E	0.11	254	309	0.97	1.30	52	21,715	1.70	13.68	5.8	0.31	166	15,340
F	0.07	156	309	0.88	1.07	74	21,715	1.29	13.68	6.2	0.46	334	15,340
G	0.07	156	379	0.84	1.20	52	15,340	1.16	14.72	6.1	0.36	325	15,340
H	0.12	146	269	1.20	0.90	74	21,715	1.24	27.30	5.6	0.82	84	15,340
I	0.07	84	269	1.38	1.10	52	21,715	0.99	41.22	5.8	0.34	217	15,340
J	0.06	64	1,227	0.84	1.00	74	5,417	0.54	33.56	4.0	0.52	144	5,417
K	0.06	60	707	1.26	0.68	103	10,834	0.92	41.22	4.0	0.27	268	10,834
L	0.06	60	269	1.10	0.58	147	30,679	0.58	47.47	5.6	0.21	479	30,680
M	0.00	7	1,070	1.40	0.75	147	5,417	0.13	232.11	1.1	0.23	403	5,417
N	0.04	5	660	3.20	0.68	210	10,834	0.20	232.11	1.9	0.14	654	10,834

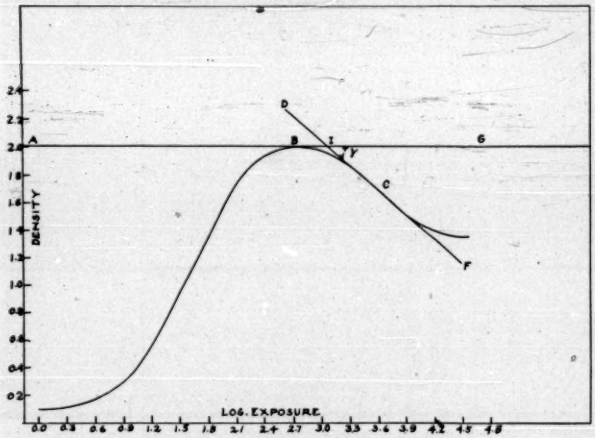
TABLE II.

Solarization Densities.

Phot. Em. Step	A	B	C	D	E	F	G	H	I	J	K	L	M	N
5	1.37	2.05	2.00	2.38	2.07	2.09	2.18	1.35	2.13	2.22	1.63	2.72	2.44	4.17
6	1.41	2.11	2.13	2.46	2.09	2.21	2.24	1.39	2.20	2.30	1.67	2.84	2.50	4.30
7	1.45	2.18	2.18	2.54	2.14	2.26	2.30	1.43	2.23	2.35	1.72	2.92	2.57	4.39
8	1.48	2.21	2.15	2.54	2.17	2.31	2.36	1.46	2.28	2.36	1.77	3.00	2.59	4.44
9	1.46	2.24	2.21	2.53	2.21	2.35	2.43	1.48	2.34	2.32	1.80	3.06	2.57	4.47
10	1.49	2.29	2.23	2.49	2.25	2.44	2.45	1.50	2.38	2.27	1.82	3.12	2.53	4.48
11	1.48	2.33	2.27	2.44	2.29	2.47	2.47	1.53	2.42	2.17	1.79	3.16	2.45	4.45
12	1.47	2.36	2.38	2.36	2.29	2.47	2.42	1.53	2.42	2.11	1.74	3.18	2.41	4.40
13	1.43	2.32	2.37	2.29	2.24	2.45	2.38	1.51	2.40	1.99	1.65	3.21	2.33	4.33
14	1.37	2.27	2.29	2.20	2.16	2.35	2.27	1.47	2.34	1.86	1.56	3.16	2.25	4.26

We may regard the fog of a photographic emulsion at its maximum density as negligible or as zero. Let an arbitrary H. and D. curve, which extends into the first reversal of the solarization, be represented in Figure 3. At the top, *B*, of

FIG. 3.



Speed measurements of the first reversal of the solarization.

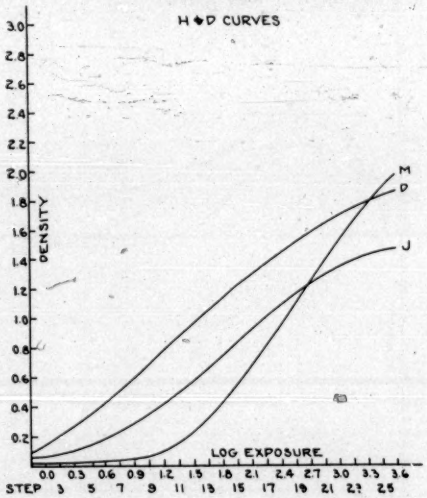
the curve is given the threshold value of the first reversal of the solarization. If a straight line *ABG* is drawn through *B*, parallel to the log *E* axis, then this line represents log *E* units of the same value as the log *E* axis. Let *C* be the inflection point of the first reversal of the solarization and *DF* the tangential line through this point, then this line will intersect *AB* at *I*. *AI* represents the inertia of the first reversal of the solarization and the H. and D. speed at the first reversal of the solarization can be expressed by:

$$\frac{10}{AI}.$$

The values obtained were so low that, for convenience, they were multiplied by 10^6. The tangent of angle *FIG* gives the

gamma of the first reversal of the solarization. Its value is negative. Both these data are given in Table I in column headed by "H.D.I sol. $\times 10^6$" and "$-\gamma$I sol." In Table II are given the densities around the threshold value of the first reversal of the solarization for the investigated photographic emulsions. In Figs. 4, 6, 8 and 10 are plotted the H. and D.

FIG. 4.

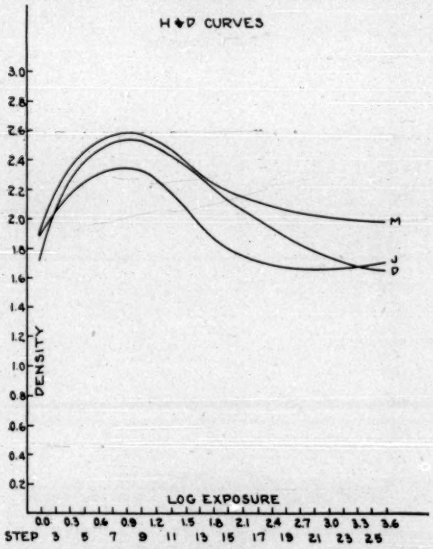


H. & D. curves.

curves, and in Figs. 5, 7, 9, and 11 the solarization curves of all the photographic emulsions investigated. The curves are so grouped that each figure, except Figs. 10 and 11, represents those emulsions which have the same threshold value of the first reversal of solarization. In Figs. 10 and 11 are the H. and D. and the solarization curves, respectively, of two emulsions which show different threshold values of the first reversal of solarization from any of the others. These threshold values range from 5,417 m.c.s. to 21,715 m.c.s. and are given in Table I.

Apart from the threshold value or its reciprocal the threshold speed and the H. and D. speed, a third way of expressing the sensitivity of a photographic emulsion has been suggested by L. A. Jones of these Laboratories. This method does not give single numbers for tabular comparison, but

FIG. 5.

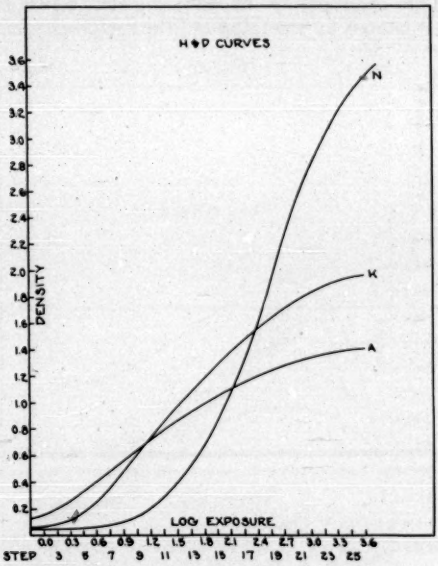


Solarization curves (of phot. emulsions DJM with t.v.I sol at 3,883 m.c.s.).

curves are obtained, which represent the rate of increase in density per unit exposure per log unit exposure by plotting D/E against log E values, in which D represents the density and E the exposure. For the solarization exposures D/E had to be multiplied by 10^3. In the normal exposure region the maximum rate of increase of density per unit exposure, $(D/E)_{max.}$, and the exposure at which this maximum occurs are characteristic of these curves. Both data are given in Table I. In the overexposure region, no characteristics could be read directly from the curves.

In Figs. 12 and 13 the D/E-log E curves for the normal and for the solarization exposures of the photographic emulsions, D, J, M are drawn. In Figs. 14 and 15, are the D/E-log E

FIG. 6.



H. & D. curves.

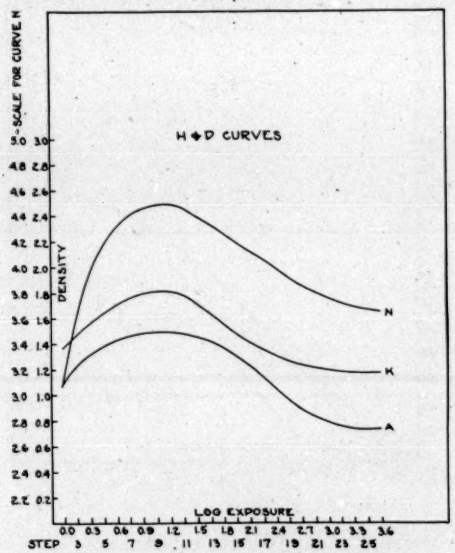
curves for the normal and for the solarization exposures of the photographic emulsions, A, N, K. In Figs. 16 and 17 are drawn the D/E-log E curves for the normal and for the solarization exposures of the photographic emulsions, F, I, B, C, E, H. In Figs. 18 and 19 are drawn the D/E-log E curves for the normal and for the solarization exposures of the photographic emulsions G and L.

GENERAL DISCUSSION OF THE DATA OBTAINED.

A comparison between the curves of Figs. 4, 6, 8 and 10 on one hand, and of Figs. 5, 7, 9 and 11 on the other, shows that

the final densities of the normal exposure region do not fit into the first densities of the overexposed region. The highest exposure in the normal exposed region is 212.98 m.c.s. and the lowest exposure in the overexposed region is 671.10 m.c.s. The discrepancy in the smooth continuation of densities is caused by the failure of the reciprocity law, owing

FIG. 7.



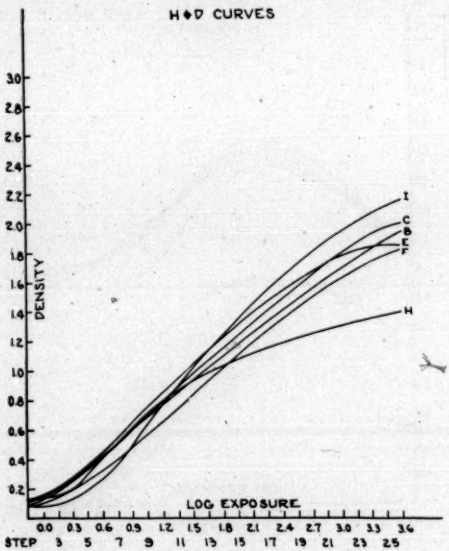
Solarization curves (of the phot. emulsions ANK with t.v.I sol at 7,670 m.c.s.).

to the application of two different light intensities during the exposures. This failure, however, will not have an appreciable influence on the required results.

In general, the threshold values of the first reversal of the solarization obtained in this investigation are smaller than Tappen's values. Without doubt the differences in the color of light play the most important part. H. Tappen used a

Welsbach mantle where we used a tungsten light. The color of the Welsbach mantle as investigated by Rubens,[9] H. E. Ives, E. F. Kingsbury and E. Karrer,[10] and M. Luckiesh [11] is

FIG. 8.



H. & D. curves.

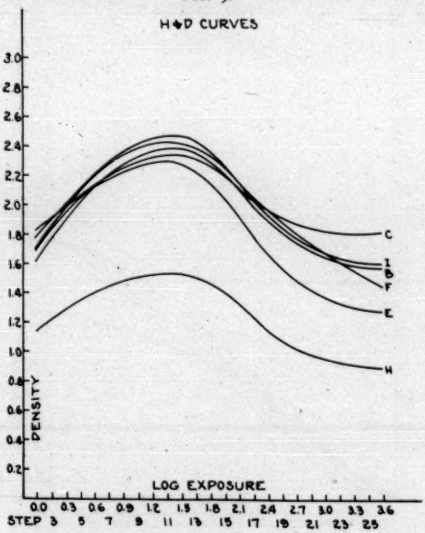
quite different from that of a gray body radiation such as tungsten.

A comparison of the curves of Figs. 4, 5, 6, 7, 8, 9, 10, and 11 seems to support Tappen's point of view that there is no direct proportionality between the threshold values of the normal exposed region and of the first reversal of the solarization region. The most pronounced case is in the curves of

[9] *Ann. Physik*, III, **20**: 593 (1906).
[10] *J. Frank. Inst.*, **186**: 401 and 585 (1918).
[11] *J. Frank. Inst.*, **183**: 633 (1917).

Figs. 6 and 7 with the photographic emulsions *A* and *N*. The threshold values for the normal exposure region are 0.052 and 0.210 m.c.s. Both have the same threshold value of the first reversal of the solarization at 10,834 m.c.s.

FIG. 9.



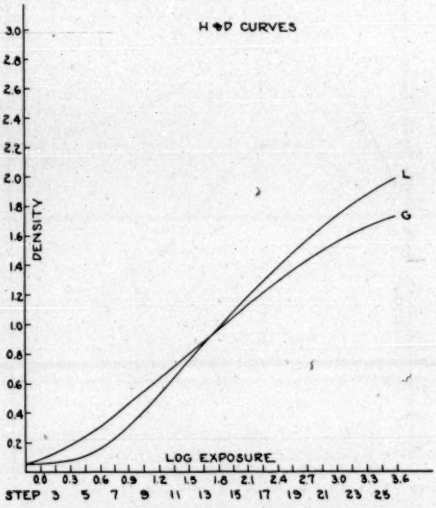
Solarization curves (of the phot. emulsions FIBCEH with t.v.I sol at 15,340 m.c.s.).

This, however, may be a deviation from a general correlation. To obtain definite proof, therefore, the subject should be treated statistically.

Figures 13, 15, 17, and 19 show a very rapid decrease in the rate of increase of the density per unit exposure in the top of the H. and D. curve and in the first reversal of the solarization. In this region the differences in the rate of increase of density per unit exposure for various photographic emulsions are much smaller than the differences in the normal exposure region for corresponding emulsions. This is in agreement with the point of view that the action of the sensitivity specks in the

overexposure region is greatly diminished. It is obvious that this is different for various photographic emulsions. It seems that the constitution and the structure of the silver halide have a dominating influence on the formation of the latent image in the overexposure and in the solarization region independent of the sensitivity speck.

FIG. 10.



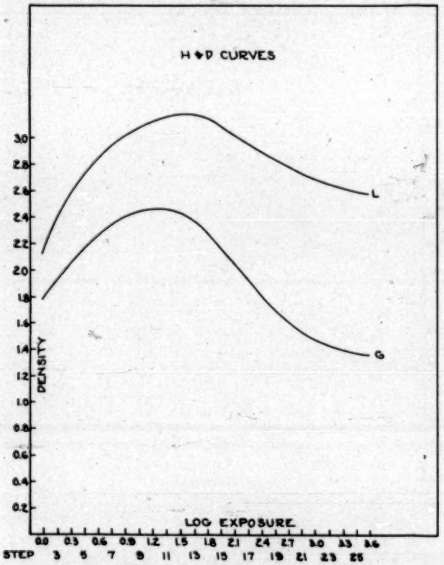
H. & D. curves (of the phot. emuls. GL with t.v.I sol at 10,834 and 21,715 m.c.s.).

THE DETERMINATION OF CORRELATIONS.

To determine statistically whether there is, or is not, a correlation between two groups of observations *a* and *b* the obtained data *a* may be arranged in increasing or in decreasing values; and it may be seen in how far the obtained data *b* in the other group increase or decrease at the same time. For instance, in Table I comparing fog with the H. and D. speeds of the various photographic emulsions, it is seen at once that with increasing H. and D. speed the fog increases also.

Only, however, if the correlation is very strong will this be possible. It is impossible to determine in this way weaker correlations. For this purpose another method is needed.[12]

FIG. 11.



Solarization curves (of the phot. emuls. GL with t.v.I sol at 10,834 and 21,715 m.c.s.).

Let us take from Table I two sets of data a and b. To find if there is a probable correlation between these two sets of data we add the values of each set separately:

$$a_1 + a_2 + a_3 + \cdots + a_n = \sum a,$$
$$b_1 + b_2 + b_3 + \cdots + b_n = \sum b,$$

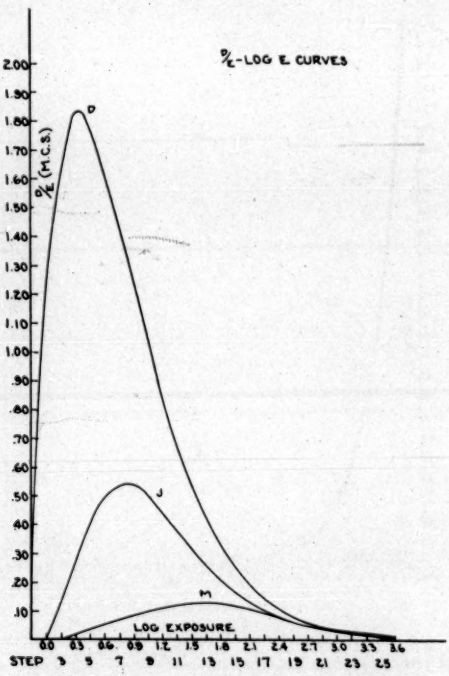
[12] *Loc. cit.* E. T. Whittaker and G. Robinson: "The Calculus of Observations," 317 (1924).

then the arithmetical mean is:

$$\bar{a} = \frac{\Sigma a}{n} \quad \text{and} \quad \bar{b} = \frac{\Sigma b}{n},$$

where n is the total number of observations in a set of data.

FIG. 12.

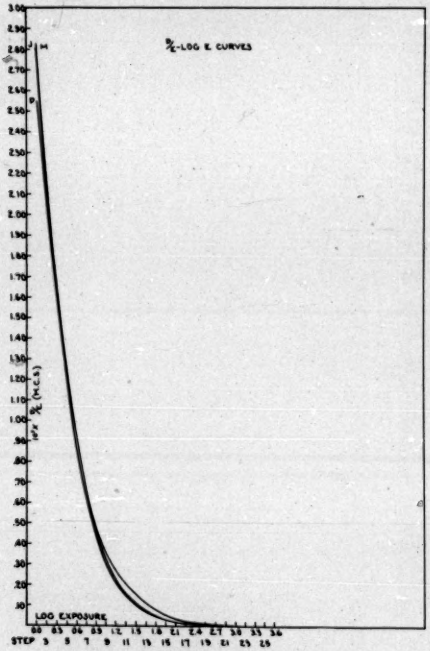


D/E curves for normal exposures of the phot. emuls. *DJM*.

Let us draw a coördinate system XOY in Fig. 20; then $x = \bar{a}$ and $y = \bar{b}$ represents a point. If this point is taken as a new origin, and new coördinates X_1, O_1, Y_1, are drawn through

it, parallel to the first ones, then the field XOY will be divided in four quadrants—I, II, III, and IV.

FIG. 13.



D/E curves for solarized exposures of the phot. emuls. DJM.

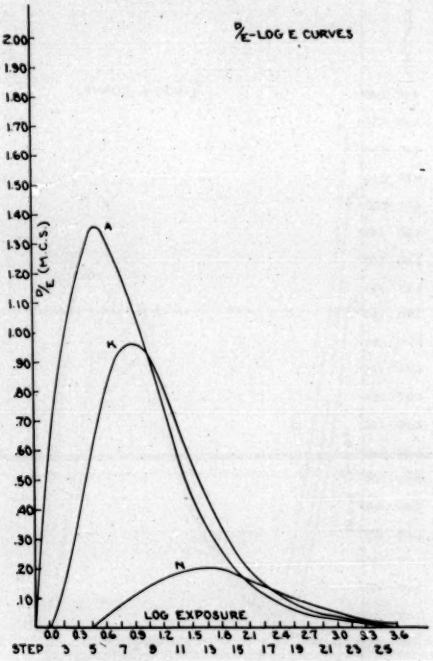
Now we determine the differences between the observed data and the arithmetical mean:

$$\xi = a - \bar{a},$$
$$\eta = b - \bar{b}.$$

These values may be positive or negative. The diagram is plotted with $\bar{a}\bar{b}$ as origin. In quadrant I are put the values

$\xi > 0$ and $\eta > 0$; in quadrant II, the values $\xi < 0$ and $\eta > 0$; in quadrant III, the values $\xi < 0$ and $\eta < 0$; and in quadrant IV, the values $\xi > 0$ and $\eta < 0$.

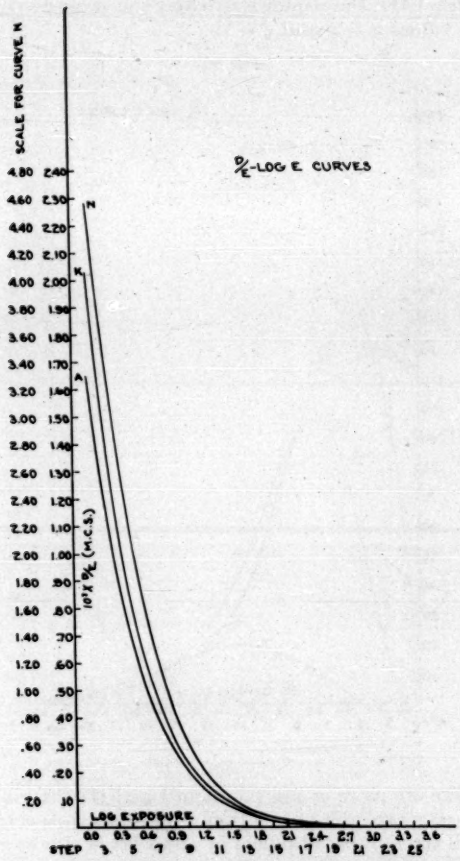
FIG. 14.



D/E curves for normal exposures of the phot. emuls. *ANK*.

If there are more or less points in I and III than in II and IV, there is a probable correlation. This probable correlation is weak if the number of points in I and III is almost equal to that of II and IV. If the numbers are equal then there is no correlation. It is obvious that the determination of the probable correlation depends on the errors in the obtained

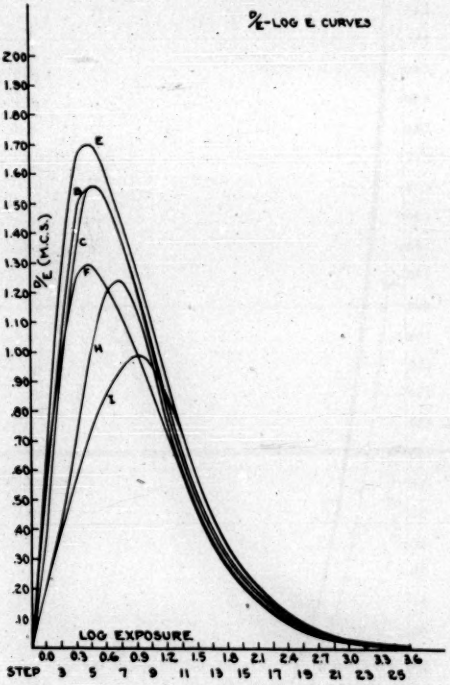
FIG. 15.



D/E curves for solarized exposures of the phot. emuls. *ANK*.

data and in the number of data. It may be, that in this way a weakly probable correlation is shown, which turns out later to be much stronger on account of the small probable error.

FIG. 16.

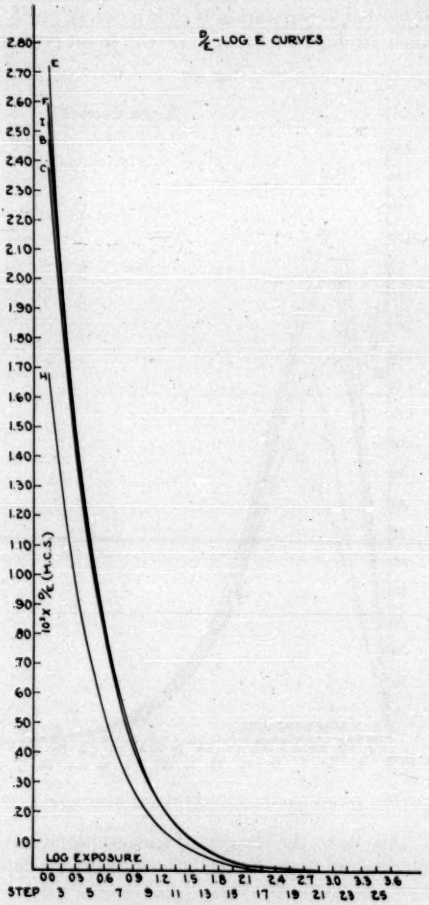


D/*E* curves for normal exposures of the phot. emuls. *FIBCEH*.

To find this out, the Bravais-Pearson coefficient k, also called the product-moment coefficient, has to be determined:

$$k = \frac{\sum \xi\eta}{\sqrt{(\sum \xi)^2 (\sum \eta)^2}}.$$

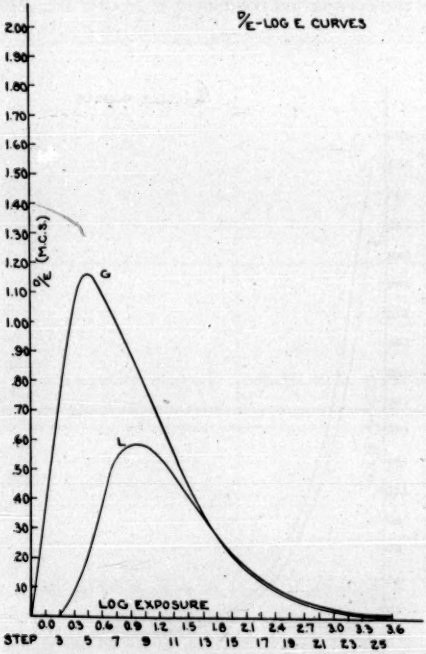
FIG. 17.



D/E curves for solarized exposures of the phot. emuls. *FIBCEH*.

If $k > 0$, then b on the whole increases with a. If $k < 0$ then b on the whole decreases with increasing a. If $k = 0$ there is no correlation; if $k = 1$, the extreme value is given, meaning direct proportionality; $k > 0$, if the total number of points

FIG. 18.



D/E curves for normal exposures of the phot. emuls. *GL*.

in the quadrants I and III is greater than the total number of points in the quadrants II and IV.

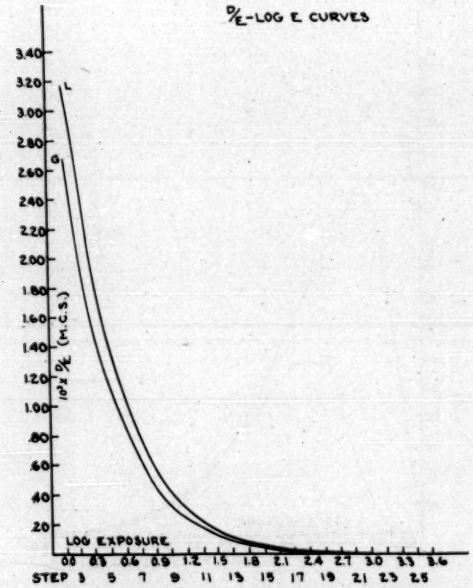
If the total number of points in the quadrants I and III is smaller than the total number of points in the quadrants II and IV, $k > 0$.

The probable error Q of the correlation coefficient is given by:

$$Q = 0.675 \frac{1 - k^2}{\sqrt{n}},$$

where n is the number of pairs of obtained data. The number of times the correlation coefficient is greater than its probable

FIG. 19.



D/E curves for solarized exposures of the phot. emuls. *GL*.

error is expressed by k/Q. If k/Q is 3 or more, we may say that there is a correlation; if k/Q is 2 or 2.5 more observations are required to determine whether or not there is a correlation; if k/Q is < 2, we may say that no definite correlation can be

found. The quantitative criterion of the correlation is given by

$$P = 1 - \theta\left(0.477\frac{k}{Q}\right),$$

which expresses the probability that the result k is due to mere chance.

FIG. 20.

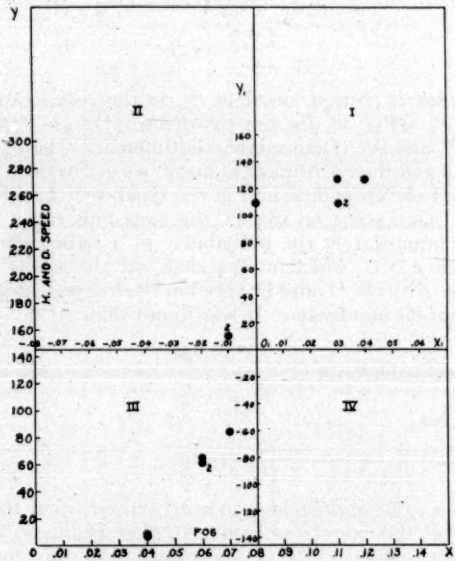


Diagram of probable correlation between H. & D. speed and fog.

If we make

$$x = 0.477\frac{k}{Q}$$

then $\theta(x)$ represents the Gaussian error function

$$\theta(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-x^2}dx.$$

The nearer $\theta(x)$ is to unity the more significant is the correlation.

Let us take as an example the fog a and the H. and D. speed b from the set of data in Table I, and let us determine quantitatively its correlation. We find:

$$\frac{\Sigma a}{14} = 0.08$$

and

$$\frac{\Sigma b}{14} = 145.0.$$

This point is plotted as origin O_1, in the coördinate system X_1, O_1, Y_1, of Fig. 20, dividing this quadrant in 4 new quadrants I, II, III, and IV. Determining the differences ξ and η between the data and their arithmetical mean, we get in the quadrants I and III eleven points; and in the quadrant II and IV, two points. One point on one of the axes falls out. This expresses immediately the probability of a strong correlation, in which $k > 0$, which means that, on the whole, the fog increases with the H. and D. speed in the fourteen investigated photographic emulsions. It was found that

$$k = 0.83,$$

with a probable error

$$Q = 0.05,$$

which gives

$$\frac{k}{Q} = 15.19.$$

P gives a value approaching so nearly to zero that the probability that this correlation is due to mere chance is less than one in a billion. The result states that the correlation approaches a direct proportionality with $k = 0.83 \pm 0.05$.

DISCUSSION OF THE CORRELATIONS OBTAINED.

The correlations found hold only for the data obtained from fourteen commercial photographic emulsions investigated, but similar sets of observations do not necessarily give the same results. The extent to which these correlations apply to all commercial products needs further investigation.

The correlations obtained give only the total result of all

TABLE III.

Correlations between speed expressions.

Region of Exposure.	No.	Correlations between:	Number of Points in Quadrant. I+III	Number of Points in Quadrant. II+IV	k.	Q.	k/Q.	P.	Sign of Corr.
Normal	1	H.D. speed: t.v.*	3	11	− 0.76	0.08	9.84	0.000,000,000···	−
	2	H.D. speed: $(D/E)_{max.}$†	14	0	0.80	0.07	12.04	0.000,000,000···	+
	3	H.D. speed: mcs. at $(D/E)_{max.}$	4	10	− 0.69	0.09	7.37	0.000,000,8	−
	4	t.v.: $(D/E)_{max.}$	2	12	− 0.70	0.09	3.64	0.000,000,2	−
	5	t.v.: mcs. at $(D/E)_{max.}$	12	2	0.86	0.05	17.67	0.000,000,000···	+
	6	$(D/E)_{max.}$: mcs. at $(D/E)_{max.}$	4	10	− 0.68	0.10	6.98	0.000,000,1	−
First Reversal of the solarization	7	H.D.I sol. speed: t.v.I sol.‡	1	13	− 0.92	0.03	32.57	0.000,000,000···	−
Normal and first reversal of the solarization	8	H.D. speed: H.D.I sol. speed	3	11	− 0.38	0.15	2.45	0.1175	?
	9	H.D. speed: t.v.I sol.	10	4	0.52	0.13	3.95	0.00784	+
	10	H.D.I sol. speed: t.v.	11	3	0.28	0.17	1.71	—	o
	11	H.D.I sol. speed: $(D/E)_{max.}$	3	11	− 0.37	0.16	2.35	0.1132	?
	12	H.D.I sol. speed: mcs. at $(D/E)_{max.}$	11	3	0.42	0.15	2.87	0.0530	+
	13	t.v.: t.v.I sol.	5	9	− 0.14	0.18	0.74	—	o
	14	t.v.I sol.: $(D/E)_{max.}$	9	5	0.23	0.09	2.56	0.0842	?
	15	t.v.I sol.: mcs. at $(D/E)_{max.}$	4	10	− 0.44	0.15	2.97	0.0451	−

* T.v. = Threshold value.

† $(D/E)_{max.}$ = maximum rate of increase of density per unit exposure.

‡ H.D.I sol. speed = H. & D. speed of the first reversal of the solarization. T.v.I sol. = Threshold value of the first reversal of the solarization.

the properties of finished photographic emulsions, and from them no direct physical meaning can always be derived. They point, rather, the direction further investigations should take. If the probable error of the observations is very small, the number of comparative experiments extensive, and no correlation found, it is useless to go on with further experiments in that direction.

In this investigation the latitude was omitted because it could not be determined accurately for most of the photographic emulsions. The correlations between the obtained sets of data in the overexposure region have been determined. We have also determined the correlations between the data obtained in this region of exposure and in the normal exposure region, but in order to secure comparable results it was necessary to reinvestigate, by means of the method described, the different sets of data in the normal exposure region. A total of 78 cases was investigated. The data were collected in Tables III, IV, and V. Where there is no correlation, a zero mark is placed in the last column. Where more data are required for the determination of an eventual correlation, a question mark is placed in the last column.

In Table III are collected the correlations between the different speed expressions. The following correlations were found:

TABLE III—a.

Region of Exposure.	No.	Correlation between	k.
Normal	(1)	H.D. speed: t.v.	-0.76 ± 0.08
	(2)	H.D. speed: $(D/E)_{max.}$	0.80 ± 0.07
	(3)	H.D. speed: m.c.s. at $(D/E)_{max.}$	-0.69 ± 0.09
	(4)	t.v.: $(D/E)_{max.}$	-0.70 ± 0.09
	(5)	t.v.: m.c.s. at $(D/E)_{max.}$	0.86 ± 0.05
	(6)	$(D/E)_{max.}$: m.c.s. at $(D/E)_{max.}$	-0.68 ± 0.10
First reversal of solarization	(7)	H.D.I sol. speed: t.v.I sol.	-0.92 ± 0.03
Normal and first reversal of solarization	(9)	H.D. speed: t.v.I sol.	0.52 ± 0.13
	(12)	H.D.I sol. speed: m.c.s. at $(D/E)_{max.}$	0.42 ± 0.15
	(15)	t.v.I sol.: m.c.s. at $(D/E)_{max.}$	-0.44 ± 0.15

There may be a correlation between:

(8) H.D. speed: H.D.I sol. speed
(11) H.D.I sol. speed: $(D/E)_{max.}$
(14) t.v.I sol.: $(D/E)_{max.}$

To be certain of this, more data are required than were obtained in this investigation.

No definite correlation could be found between

(10) H.D.I sol. speed: t.v.
(13) t.v.: t.v.I sol.

This last result confirms the statements of Walter and Tappen, mentioned previously.

If we look over the numbers in Table III, we see that in general the correlation coefficients are much higher both in the normal exposure region and in the first reversal of the solarization than the correlations between the speed expressions in these two regions of exposure. The probable error of the correlation coefficient is also much smaller and the probability that the result is mere chance is very much less. Even where three correlations are found definitely between the two regions of exposures, two of them are rather weak, but worthy of consideration.

In Table IV are presented the correlation investigations between different photographic characteristics and between these characteristics and the speed expressions. They are classified according to the regions of exposure: normal; at maximum density and the first reversal of the solarization; and between these regions.

TABLE IV.

Correlations between different photographic characteristics.

Region of Exposure.	No.	Correlations between:	Number of Points in Quadrant.		k.	Q.	k/Q.	P.	Sign of Corr.
			I+III	II+IV					
Normal	16	fog: H.D. speed	11	2	0.83	0.05	15.19	0.000,000,000···	+
	17	fog: γ *	5	8	− 0.48	0.14	3.49	0.1890	−
	18	fog: t.v.	4	9	− 0.70	0.09	7.55	0.000,000,2	−
	19	fog: $(D/E)_{max.}$	11	2	0.70	0.09	7.42	0.000,000,2	+
	20	fog: mcs. at $(D/E)_{max.}$	6	7	− 0.75	0.08	9.72	0.000,000,000···	−
	21	H.D. speed: γ	4	10	− 0.55	0.12	4.42	0.002,85	−
	22	γ: t.v.	10	4	0.79	0.07	11.80	0.000,000,000···	+
	23	γ: $(D/E)_{max.}$	4	10	− 0.48	0.14	3.46	0.01962	−
	24	γ: mcs. at $(D/E)_{max.}$	10	4	0.77	0.07	10.57	0.000,000,000···	+
At $D_{max.}$	25	mcs. at $D_{max.}$: $D_{max.}$	6	8	0.11	0.18	0.59	—	o
First reversal of solarization	26	H.D.I sol.: − γ sol.†	3	9	− 0.29	0.16	1.76	—	o
	27	− γI sol.: t.v.I sol.	9	3	0.22	0.17	1.30	—	o
Normal and at $D_{max.}$	28	$D_{max.}$: fog	3	10	− 0.54	0.13	4.28	0.00391	−
	29	$D_{max.}$: H.D. speed	6	8	− 0.48	0.14	3.41	0.02116	−
	30	$D_{max.}$: γ	6	8	0.75	0.08	9.31	0.000,000,000···	+
	31	$D_{max.}$: t.v.	10	4	0.73	0.09	8.71	0.000,000,000···	+
	32	$D_{max.}$: $(D/E)_{max.}$	6	8	− 0.44	0.15	3.04	0.0403	−
	33	$D_{max.}$: mcs. at $(D/E)_{max.}$	10	4	0.63	0.11	5.86	0.000,08	+
	34	mcs. at $D_{max.}$: fog	8	5	0.34	0.16	2.15	0.1468	~
	35	mcs. at $D_{max.}$: H.D. speed	11	3	0.46	0.14	3.28	0.02737	+
	36	mcs. at $D_{max.}$: γ	6	8	− 0.15	0.18	0.84	—	o
	37	mcs. at $D_{max.}$: t.v.	8	6	0.25	0.17	1.45	—	o
	38	mcs. at $D_{max.}$: $(D/E)_{max.}$	10	4	0.13	0.18	0.74	—	o
	39	mcs. at $D_{max.}$: mcs. at $(D/E)_{max.}$	4	10	− 0.31	0.16	1.91	—	o

TABLE IV.—*Continued.*

Region of Exposure.	No.	Correlations between:	Number of Points in Quadrant. I+III	Number of Points in Quadrant. II+IV	k.	Q.	k/Q.	P.	Sign of Corr.
Normal and first reversal of the solarization	40	fog: H.D.I sol. speed	4	9	− 0.62	0.17	3.71	0.001 231	−
	41	fog: $-\gamma$I sol.	6	5	0.33	0.16	2.06	0.1645	?
	42	fog: t.v.I sol.	9	4	0.42	0.15	2.84	0.065	+
	43	H.D. speed: $-\gamma$I sol.	8	4	0.36	0.16	2.26	0.1274	?
	44	H.D.I sol. speed: γ	9	5	0.12	0.18	0.69	—	0
	45	γ: $-\gamma$I sol.	5	7	− 0.25	0.17	1.50	—	0
	46	γ: t.v.I sol.	7	7	0.15	0.18	0.86	—	0
	47	$-\gamma$I sol.: t.v.	2	10	− 0.12	0.11	1.30	—	0
	48	$-\gamma$I sol.: $(D/E)_{max.}$	7	5	0.08	0.18	0.44	—	0
	49	$-\gamma$I sol.: mcs. at $(D/E)_{max.}$	4	8	− 0.43	0.15	2.92	0.0490	−
At $D_{max.}$ and first reversal of the solarization	50	$D_{max.}$: H.D.I sol. speed	9	5	0.16	0.18	0.94	—	0
	51	$D_{max.}$: $-\gamma$I sol.	4	8	− 0.27	0.17	1.62	—	0
	52	$D_{max.}$: t.v.I sol.	5	9	0.03	0.18	0.02	—	0
	53	mcs. at $D_{max.}$: H.D.I sol. speed	1	13	− 0.77	0.07	10.34	0.000,000,000···	−
	54	mcs. at $D_{max.}$: $-\gamma$I sol.	12	2	0.15	0.18	0.84	—	0
	55	mcs. at $D_{max.}$: t.v.I sol.	13	1	0.92	0.03	34.11	0.000,000,000···	+

* γ = gamma of the normal exposed region.
† $-\gamma$I sol. = gamma of the H. & D. curve in the first reversal of the solarization.

We find in the normal exposure region the following definite correlations:

TABLE IV—a.

Region of Exposure.	No.	Correlation between	k.
Normal	(16)	fog: H.D. speed	0.83 ± 0.05
	(17)	fog: γ	-0.48 ± 0.14
	(18)	fog: t.v.	-0.70 ± 0.09
	(19)	fog: $(D/E)_{max.}$	0.70 ± 0.09
	(20)	fog: m.c.s. at $(D/E)_{max.}$	-0.75 ± 0.08
	(21)	H.D. speed: γ	-0.55 ± 0.12
	(22)	γ: t.v.	0.79 ± 0.07
	(23)	γ: $(D/E)_{max.}$	-0.48 ± 0.14
	(24)	γ: m.c.s. at $(D/E)_{max.}$	0.77 ± 0.07

As stated above, the correlations are not always of direct physical significance. This is plainly shown in correlation (16). In the laboratory it is possible to increase the fog of an experimental emulsion and, at the same time, to decrease its H. and D. speed which would contradict the correlation expressed in (16). A diluted solution of methylene blue would be sufficient to show this simultaneous increase in fog and decrease in H. and D. speed with every photographic emulsion. With commercial emulsions it is necessary to keep fog as low as possible, even if the highest H. and D. speed has to be reached. Under the conditions of development to obtain the fog densities of Table I we may say that these values represent inherent emulsion fog.

At the maximum density we do not find any correlation between:

(25) m.c.s. at $D_{max.}$: $D_{max.}$

A deviation of the rule is noted here. In spite of the fact that the total number of points in quadrants I and III is smaller than the total number of points in quadrants II and IV, which should give a negative correlation coefficient, this coefficient is positive. This is due to the small difference between the total numbers in the two groups of quadrants and the extraordinarily high value of the maximum density of the photographic emulsion N, which overbalances the values I and III.

In the first reversal of the solarization no correlation is found between:

(26) H.D. speed: γ
(27) γ: t.v.

Between the photographic characteristics and the speed expressions at the normal exposure region at one side and those at the maximum density at the other side, we find the following correlations:

TABLE IV—*b*.

Region of Exposure.	No.	Correlations between	*k*.
Normal and at $D_{max.}$	(28)	$D_{max.}$: fog	-0.54 ± 0.13
	(29)	$D_{max.}$: H.D. speed	-0.48 ± 0.14
	(30)	$D_{max.}$: γ	0.75 ± 0.08
	(31)	$D_{max.}$: t.v.	0.73 ± 0.09
	(32)	$D_{max.}$: $(D/E)_{max.}$	-0.44 ± 0.15
	(33)	$D_{max.}$: m.c.s. at $(D/E)_{max.}$	0.63 ± 0.11
	(35)	m.c.s. at $D_{max.}$: H.D. speed	0.46 ± 0.14

More observations are required to determine whether or not there is a definite correlation between:

(34) m.c.s. at $D_{max.}$: fog

No definite correlation could be found between:

(36) m.c.s. at $D_{max.}$: γ
(37) m.c.s. at $D_{max.}$: t.v.
(38) m.c.s. at $D_{max.}$: $(D/E)_{max.}$
(39) m.c.s. at $D_{max.}$: m.c.s. at $(D/E)_{max.}$

Correlation (30) gives a positive correlation coefficient in spite of the fact that the total number of points in quadrant I and III is smaller than the total number of points in quadrants II and IV. This case is similar to the one mentioned before for correlation (25) and the same explanation holds for correlation (30).

In general, in this group of data the maximum density shows correlations with various photographic characteristics and speed expressions. This is not the case with the exposure needed to reach the maximum density: correlation (35) has a low correlation coefficient.

Between the photographic characteristics and the speed expressions in the normal exposure region on the one hand, and those of the first reversal of the solarization on the other, we find the following correlations:

TABLE IV—*c*.

Region of Exposure.	No.	Correlations between	*k*.
Normal and first reversal of solarization	(40)	fog: H.D.I sol. speed	-0.62 ± 0.17
	(42)	fog: t.v.I sol.	0.42 ± 0.15
	(49)	γI sol.: m.c.s. at $(D/E)_{max.}$	-0.43 ± 0.15

Data, further than those given in Table I, are required to determine whether or not there is a correlation between:

(41) fog: $-\gamma$I sol.
(43) H.D. speed: $-\gamma$I sol.

No definite correlations could be found between:

(44) H.D.I sol. speed: γ
(45) γ: $-\gamma$I sol.
(46) γ: t.v.I sol.
(47) $-\gamma$I sol.: t.v.
(48) $-\gamma$I sol.: $(D/E)_{max.}$

The correlations (42) and (43) are rather weak but worthy of consideration.

Between the photographic characteristics and the speed expression at the maximum density on the one hand and those of the first reversal of the solarization on the other, we find the following correlations:

TABLE IV—*d*.

Region of Exposure.	No.	Correlations between	*k*.
At $D_{max.}$ and first reversal of solarization	(53)	m.c.s. at $D_{max.}$: H.D.I sol. speed	-0.77 ± 0.07
	(55)	m.c.s. at $D_{max.}$: t.v.I sol.	0.92 ± 0.03

TABLE V.

Correlations between the average grain size and the iodide content and different photographic characteristics.

Region of Exposure.	No.	Correlations between:	Number of Points in Quadrant. I+III	Number of Points in Quadrant. II+IV	k.	Q.	k/Q.	P.	Sign of Corr.
Normal	56	fog: % AgI	9	4	0.74	0.08	8.94	0.000,000,000···	+
	57	fog: average grain size	10	3	0.71	0.09	7.79	0.000,000,2	+
	58	H.D. speed: % AgI	11	3	0.71	0.09	7.85	0.000,000,8	+
	59	H.D. speed: average grain size	10	4	0.51	0.13	3.82	0.010,06	+
	60	γ: % AgI	5	9	− 0.64	0.11	6.02	0.000,04	−
	61	γ: average grain size	4	10	− 0.53	0.13	4.07	0.006,08	−
	62	t.v.: % AgI	3	11	− 0.76	0.08	9.82	0.000,000,000···	−
	63	t.v.: average grain size	4	10	− 0.60	0.11	5.25	0.000,41	−
	64	$(D/E)_{max.}$: % AgI	11	3	0.66	0.10	6.49	0.000,02	+
	65	$(D/E)_{max.}$: average grain size	10	4	0.40	0.15	2.66	0.0729	?
	66	mcs. at $(D/E)_{max.}$: % AgI	3	11	− 0.89	0.04	24.01	0.000,000,000···	−
	67	mcs. at $(D/E)_{max.}$: average grain size	6	8	− 0.54	0.13	4.28	0.00391	−
	68	% AgI: average grain size	9	5	0.50	0.13	3.71	0.001,231	+
At $D_{max.}$	69	$D_{max.}$: % AgI	5	9	− 0.43	0.15	2.90	0.0501	−
	70	$D_{max.}$: average grain size	4	10	− 0.75	0.08	9.54	0.000,000,000···	−
	71	mcs. at $D_{max.}$: % AgI	13	1	0.62	0.11	5.50	0.000,21	+
	72	mcs. at $D_{max.}$: av. grain size	8	6	0.046	0.18	0.25	—	0
First reversal of the solarization	73	H.D.I sol. speed: % AgI	0	14	− 0.72	0.09	8.25	0.000,000,000···	−
	74	H.D.I sol. speed: av. grain size	5	9	− 0.28	0.17	1.66	—	0
	75	− γI sol.: % AgI	11	3	0.49	0.14	3.53	0.0175	+
	76	− γI sol.: av. grain size	8	4	0.40	0.15	2.59	0.0795	?
	77	t.v.I sol.: % AgI	12	2	0.67	0.10	6.78	0.000,001	+
	78	t.v.I sol.: av. grain size	9	5	0.14	0.18	0.80	—	0

No correlation was found between:

(50) $D_{max.}$: H.D.I sol. speed
(51) $D_{max.}$: $-\gamma$I sol.
(52) $D_{max.}$: t.v.I sol.
(54) m.c.s. at $D_{max.}$: $-\gamma$I sol.

The strong correlation (55) is very obvious: The maximum density of all the commercial emulsions investigated extends over two or three steps and shows in many cases so sharp a peak that the values of the exposures necessary to reach the maximum density are practically identical with the threshold values of the first reversal of the solarization. The same thing makes correlation (53) equal to (7); (25) to (52); (34) to (42); (35) to (9); (36) to (46); (37) to (13); (38) to (14); and (39) to (15). The differences in correlation coefficients are due only to the flatness of the top of the curve.

If we take a general view of Table IV we find the same thing that was found in Table III. The correlations between the data in the normal exposure region are strong with very low P values, and the correlations between the data of the normal exposure region and the data of the first reversal of the solarization are weak or do not exist. There are no correlations between the photographic characteristics in the solarization region. There are definite correlations between the maximum density and the photographic characteristics of the normal exposure region, but there are no correlations between the photographic characteristics of the first reversal of the solarization and the maximum density.

In Table V are collected the investigations of the correlations between the average grain size, the iodide content of the silver halide, various photographic characteristics and the speed expressions for the normal region of exposure, at the maximum density, and in the region of the first reversal of the solarization.

In the normal exposure region the following correlations are found:

TABLE V—*a*.

Region of Exposure.	No.	Correlations between	*k*.
Normal	(56)	fog: % AgI	0.74 ± 0.08
	(57)	fog: average grain size	0.71 ± 0.09
	(58)	H.D. speed: % AgI	0.71 ± 0.09
	(59)	H.D. speed: average grain size	0.51 ± 0.13
	(60)	γ: % AgI	− 0.64 ± 0.11
	(61)	γ: average grain size	− 0.53 ± 0.13
	(62)	t.v.: % AgI	− 0.76 ± 0.08
	(63)	t.v.: average grain size	− 0.60 ± 0.11
	(64)	$(D/E)_{max.}$: % AgI	0.66 ± 0.10
	(66)	m.c.s. at $(D/E)_{max.}$: % AgI	− 0.89 ± 0.04
	(67)	m.c.s. at $(D/E)_{max.}$: average grain size	− 0.54 ± 0.13
	(68)	% AgI: average grain size	0.50 ± 0.13

No determination of a definite correlation could be found between:

(65) $(D/E)_{max.}$: average grain size.

It is probable that there is a correlation with a positive correlation coefficient; to determine this definitely, more observations are needed.

Again, we find here pronounced examples confirming the statement that the correlations are only total results, and it is evident, from correlations (56) to (68), that they cannot always present direct physical meaning without further knowledge concerning them. It is a well known fact that silver iodide is formed in a photographic emulsion to prevent fog. This is contrary to correlation (56). In diminishing fog, there is an opportunity to increase the sensitivity of a photographic emulsion to higher values. Correlation (56), which has a strong correlation coefficient, is the result of this method of producing a minimum inherent emulsion fog. Correlation (68) expresses the same result found by F. F. Renwick and V. B. Sease [13] and which was confirmed by S. E. Sheppard and one of us [14] that the larger silver bromo-iodide grains contain more silver iodide than the smaller grains. In spite of this fact, correlation (68) has a quite different meaning. It is well known that with increasing iodide content of a potassium bromo-iodide mixture with gelatin and silver nitrate, a decreasing average grain size precipitate of silver

[13] *Phot. J.*, **64**: 360 (1924).

[14] *J. Frank. Inst.*, **203**: 829 (1927).

bromo-iodide is produced. A. P. H. Trivelli and S. E. Sheppard,[15] B. D. Baldsiefen, V. B. Sease and F. F. Renwick,[16] A. Steigmann,[17] and Lüppo-Cramer [18] have called attention to this fact. To increase the possibility of obtaining higher sensitivity of a photographic emulsion the ripening has to be extended or special precipitation conditions have to be applied for the formation of larger grains. With an increase of the silver iodide content of the silver bromo-iodide grain in addition to the specific effect of the iodide on sensitiveness the chance of obtaining emulsion fog diminishes, and the ripening can be carried on for the formation of larger grains, resulting in the correlation expressed in (68).

The following correlations are found at the maximum density:

TABLE V—*b*.

Region of Exposure.	No.	Correlations between	*k*.
At $D_{max.}$	(69)	$D_{max.}$: % AgI	-0.43 ± 0.15
	(70)	$D_{max.}$: average grain size	-0.75 ± 0.08
	(71)	m.c.s. at $D_{max.}$: % AgI	0.62 ± 0.11

There is no correlation between:

(72) m.c.s. at $D_{max.}$: average grain size

In the first reversal of the solarization we find the correlations:

TABLE V—*c*.

Region of Exposure.	No.	Correlations between	*k*.
First reversal of solarization	(73)	H.D.I sol. speed: % AgI	-0.72 ± 0.09
	(75)	$-\gamma$I sol.: % AgI	0.49 ± 0.14
	(77)	t.v.I sol.: % AgI	0.67 ± 0.10

No definite correlation could be found between (76) $-\gamma$I sol.: average grain size.

[15] "The Silver Bromide Grain." Monographs on the Theory of Photography, Eastman Kodak Co., Rochester, N. Y., 1921.

[16] *Phot. J.*, **66**: 163 (1926).

[17] *Phot. Ind.* 228 (1925).

[18] *Z. wiss. Phot.* **27**: 9 (1929).

More observations are necessary to determine definitely whether or not there is correlation here.

No correlation exists between:

(74) H.D.I sol. speed: average grain size
(78) t.v.I sol.: average grain size.

The correlations (73) and (77) are in agreement with Schumann and Tappen's statement, that there is an influence of the silver iodide content of the silver halide on the solarization sensitivity. But, where they believed that the solarization speed would increase with increasing silver iodide content, this statistical investigation shows the opposite. In general it is regarded that silver iodide solarizes sooner than silver bromide, which is not in agreement with the sign of the correlation coefficient of the correlations (73) and (77). In both these correlations the value of P is small, which emphasizes the reliability of the result. We must conclude, therefore, that there is a factor overlapping the influence of the silver iodide in the silver halide forcing the total result in an opposite direction. The nature of this factor will be discussed in *Conclusions*.

Again, comparing all the correlations of Table V: strong correlations are found in the normal exposure region and even at the maximum density. But in the first reversal of the solarization it is different. There is a strong correlation with the composition of the silver halide and a weak or no correlation with the average grain size of the silver halide. Comparing two photographic emulsions, Lüppo-Cramer [19] detected recently that the grain size of the silver halide had no influence on the threshold value of the first reversal of the solarization.

CONCLUSIONS.

The correlations give valuable information for the theory of the latent image. Nearly everywhere in the normal exposure region are strong correlations between average grain size of the silver halide, the maximum density, the contrast, the Hurter and Driffield speed, the threshold value, the rate of increase of density per unit exposure, *etc.* The equivalent correlations do not exist between the solarization region of

[19] *Z. wiss. Phot.*, **27**: 9 (1929).

exposures and the normal exposure region or if a few of them hold, they are weak.

From the standpoint of the concentration speck theory as it was first suggested by S. E. Sheppard, A. P. H. Trivelli, and R. P. Loveland [20] and later placed on a photo-electric basis by one of us,[21] we may say that this theory of the latent image is in agreement with the correlations found in the normal exposure region. The concentration speck theory in this form explains the action of the sensitivity speck as one collecting silver on an electrode through electrolysis of the silver halide forming a photo-electric Becquerel cell of the constitution,

$$Ag\,|\,Ag\ Hal\,|\,Ag_2S,$$

by which the silver cathode gradually envelops the silver sulfide anode. The existence of silver sulfide in the sensitivity speck has been discovered by S. E. Sheppard.[22] It is obvious that in such a cell the action is limited. In a previous investigation on the action of chromic acid on the sensitivity of solarized silver bromide, by R. P. Loveland and one of us, it was concluded that the sensitivity specks exhaust their action as collectors of silver atoms in the overexposure region. The concentrating action of the specks approaches low values at the shoulder of the Hurter and Driffield curve and disappears in the region of the first reversal of the solarization. This differs with photographic emulsions. The absence of correlations (or if correlations exist they are weak) between the photographic characteristics in the normal exposure region and in the first reversal of the solarization agrees with and supports the conclusion that there is evidence for the limited action of the sensitivity speck, which is according to theoretical requirement. The concentration action of the Becquerel cell of the speck theory has ceased its work nearly completely in the solarization region of exposures. But the existence of a few weak correlations between the normal exposure region and the first reversal of the solarization suggests that the sensitivity speck has left some influence on the photographic characteristics of the first reversal of the solarization as is expressed in the correlations (40) and (42). There is also a

[20] *J. Frank. Inst.*, **200**: 51 (1925).
[21] *J. Frank. Inst.*, **204**: 649 (1927); **205**: 111 (1928).
[22] Coll. Symp. Monograph **3**: 76 (1925). *Phot. J.*, **65**: 380 (1925).

weak correlation between the Hurter and Driffield speed of the normal exposure region and the threshold value of the first reversal of the solarization as expressed in correlation (9).

In an investigation by S. E. Sheppard, E. P. Wightman, and one of us [23] and by W. Clark [24] it was shown that in a highly sensitive emulsion a treatment with chromic acid affected the larger grains more than the smaller grains. This led to the conclusion that the various grains contain, in all probability, sensitivity specks of different size. The distribution of these different sizes should be regarded as being so controlled by the laws of probability that the largest grains have the greatest chance to contain the largest sensitivity specks. If the sensitivity speck is too large, then it is a center of developability producing inherent emulsion fog as pointed out by one of us, E. P. Wightman and S. E. Sheppard.[25] The emulsion fog decreases on the whole the speed of the first reversal of the solarization, which suggests the conclusion that it takes a greater exposure to exhaust the action of the larger sensitivity specks than the action of the smaller specks. The larger sensitivity specks extend their action of collecting silver atoms toward larger exposures than the smaller sensitivity specks until within the first reversal of the solarization, shifting the appearance of solarization toward higher exposures. *The sensitivity speck removes solarization.* This is in agreement with K. C. D. Hickman's [26] theory of the action of the silver sulfide of the sensitivity speck as a halogen absorber, free silver being formed. The presence of a halogen absorber prevents solarization. In the investigation of one of us with R. P. Loveland, in which it is explained how chromic acid may increase the developability in the overexposure and in the solarization region, two hypotheses are proposed. The first one was that in its action on gelatin chromic acid formed a decomposition product which was a halogen absorber and removed solarization. The second one was that new sensitivity specks were formed, which showed their action first in the overexposure region. According to these conclusions, we may regard both hypotheses as one and the same, if the

[23] *Trans. Farad. Soc.*, 19: 309 (1923); *J. Frank. Inst.*, 196: 653 and 779 (1923).
[24] *Phot. J.*, 64: 91 (1924).
[25] *Phot. J.*, 65: 134 (1925).
[26] *Phot. J.*, 67: 34 (1927).

sensitizing produced by the action of chromic acid on gelatin is due to a silver sulfide-containing speck. But then the question arises: Why does this speck show its first actions in such an over-exposure region?

The sensitivity speck formed by the making of the photographic emulsion is different from that formed by chromic acid and gelatin. This difference may be due to differences in resistance of the concentration action, caused by a different adsorption from the silver sulfide on the silver bromide or a different compactness of the silver sulfide of the speck. Another reason may be that the original sensitivity specks present on the silver bromide crystals before the chromic acid treatment form a complete Becquerel cell. In this cell the later formed sensitivity specks are only silver sulfide specks on silver bromide, which need first a contact with photochemically formed silver at the boundary between the silver sulfide and the silver bromide to form a Becquerel cell, which then will work as a concentration speck for silver atoms.

In all these cases the conclusion drawn from the statistical investigation of correlation is in agreement with the suggested hypotheses to explain the increase of developability in the overexposure region by the chromic acid reaction.

This investigation may give some indication of the cause of the first reversal of the solarization. At the moment there are two theories proposed by two groups of investigators. In one group there are: Lüppo-Cramer,[27] M. Andresen,[28] J. M. Eder,[29] C. E. K. Mees,[30] and H. Kieser,[31] who regard the action of liberated halogen from the silver halide of the grain on the centers of development as the cause of the first reversal of the solarization. In the second group are J. Eggert and W. Noddack,[32] H. Arens,[33] H. Scheffers,[34] W. Leszynski,[35] and W. Meidinger,[36] who regard the cause of the first reversal of the

[27] *Phot. Korr.*, 376 and 439 (1907); *Das latente Bild* (1911); *Phot. Ind.*, **26**: 596 and 960 (1928); *Zeitschr. wiss. Phot.*, **25**: 224 and 308 (1928); *Eder's Handb.*, **2** : 600 (1927).

[28] *Das latente Lichtbild*, 36 (1913).

[29] *Zeitschr. phys. Chem.*, **117**: 293 (1925); *Eder's Handb.*, **2I**: 585 (1927).

[30] *Abridged Sci. Pub.*, **1**: 74 (1913/14).

[31] *Z. wiss. Phot.*, **26**: 321 (1929).

[32] *Eder's Handb.*, **2I**: 246 (1927).

[33] *Z. physik. Chem.*, **114**: 337 (1925).

[34] *Z. Physik*, **20**: 109 (1929).

[35] *Z. wiss. Phot.*, **24**: 282 (1926); *Phot. Ind.*, 425 (1928).

[36] *Handb. phys. Optik*, 50 (1927).

solarization as a coagulation of the photolytically formed silver on the surface of the silver halide grain.

This theory requires a correlation between the average grain size and the sensitivity of the first reversal of the solarization, while the first theory requires a correlation between the composition of the silver halide grain and the sensitivity of the first reversal of the solarization. We could not find any correlation between the average grain size and the sensitivity of the first reversal of the solarization, but we found a definite correlation between the silver iodide content of the silver halide and the sensitivity of the first reversal of the solarization.

From a statistical point of view we may conclude that the regression theory of solarization is in better agreement with the correlations found than is the coagulation theory.

The conclusions we have made about the action of the sensitivity speck in the solarization region requires a correction about the action of the silver iodide content on the sensitivity of the first reversal of the solarization. Tappen found an increase of sensitivity and the correlation gives a decrease of sensitivity of the first reversal of the solarization by increasing silver iodide content of the silver halide. We have here the resultant of two competing actions produced by the fact that the silver iodide content of the grains and the sensitivity speck act in reverse order on the sensitivity of the first reversal of the solarization and the dominating influence of one of the factors reverses the sign of the correlation. We may conclude with a complete affirmation of H. Tappen's conclusions.

SUMMARY.

1. A statistical investigation was made for the determination of correlations between different photographic characteristics in the normal exposure region, at the maximum density and in the first reversal of the solarization from 14 different commercial photographic emulsions.

2. The investigation was made quantitatively by determining the Bravais-Pearson correlation coefficient, its probable error, and the criterion in how far the result was due to mere chance.

3. Based on the concentration speck theory as the action of an amicroscopic Becquerel cell, the consequences for this theory were drawn from the determined correlations.

Spectral Reflectances of Common Materials in the Ultraviolet Region. M. LUCKIESH. (*J. Optical Soc. of America and Rev. of Scientific Instruments*, July, 1929.) Because of the interest now shown in the relation of ultraviolet radiation to health it is well to examine the properties of materials in respect to transmitting and reflecting the shorter wave-lengths. Data from different sources are collated and presented in the form of curves.

Bleached cotton reflects a larger part of ultraviolet radiation than do linen, bleached wool and pongee silk. "Therefore, for protection against ultraviolet radiation outdoors and particularly in the tropics white cotton should be appreciably superior to other common fabrics." Among pigments zinc oxide has a very low reflectance. For example at wave-length 3600 Å. it reflects only 2 per cent. while white lead reflects 73 per cent. This explains the marked difference observed when adjacent areas covered respectively with white lead and zinc oxide are photographed. "In cases where a high reflectance is desirable in the visible region and absorption of ultraviolet is necessary zinc oxide is particularly attractive." Dry powders of aluminium oxide, magnesium carbonate, slaked lime, kaolin and wall plaster reflect rather well in the ultraviolet. Bronze paint reflects little but aluminium reflects well. White duco reflects almost no ultraviolet radiation. Silver reflects a large proportion of visible radiation but for the ultraviolet its reflectance decreases from the visible region and reaches a minimum of 7 per cent. for wave-length 3100 Å. For shorter waves its reflectance again increases.

"When artificial lighting practice reaches the stage of combining with lighting for seeing specific radiations for the preservation of health, spectral reflecting characteristics of materials will be of great importance. The data pertaining to materials which have been presented herewith are already of utmost importance in outdoor life and in special uses of ultravoilet radiation from artificial sources."

G. F. S.

UNBRACED CABLES.

BY

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SYNOPSIS

For long span cables, either with or without concentrated load, the usual methods of design are found to be inaccurate because they omit the elastic stretch of the cable and they assume an inaccurate shape of the resulting curve. Equations are derived to cover all possible cases, loaded and unloaded cable, based upon assumptions of elastic flexible cables with fixed supports. Equations are also derived showing the variation in several quantities with changes in temperature and movement of supports. Conditions of possible loads and elastic properties of cables are discussed. A bibliography covering major references to the subject is included.

A. THE SYMMETRICAL CATENARY WITH HORIZONTAL SUPPORTS.

A perfectly flexible string of no weight, an imaginary condition equivalent to a geometric line, when suspended from two fixed points at the same elevation takes the shape of a curve called a catenary. The actual case under discussion is the curve assumed by a uniform wire or cable when suspended from two fixed points. Such cable has weight and the effect of such weight is to give an elastic stretch to the wire. The resulting curve is not a catenary and if it is assumed that the cable is perfectly elastic and that the tensions in the cable do not exceed the elastic limit of the material, the curve assumed by the cable is called the elastic catenary.

The elastic catenary curve has a larger sag than the catenary, which latter has a greater maximum slope than a parabolic curve hung from the same fixed supports. The usual approximation is that the curve taken by a suspended cable is a parabola. Since the actual sag is greater than that which results from such an assumption, the maximum tension is less and as a result, a careful analysis of a cable will show, in some cases, as high as 10 per cent. greater strength than that shown by the approximate method.

A cable hung between two fixed points takes its shape because of the gravitational action on its weight and, assuming

that the cable is perfectly flexible, the slope of the curve of any point is the slope of the tension at that point. From this condition can be derived the equation of the true shape of the curve taken.

Another condition which can be used and which gives the same results is that the center of gravity of the cable weight must assume its lowest possible position under the induced internal stresses.

The derivation of the equations showing the relations between the horizontal and vertical coördinates and also the angle of deviation from the horizontal is given in the mathematical tabulation in Table (*a*).

TABLE (*a*)

UNBRACED CABLES (a)

HORIZONTAL SPAN.
(FIG. 1.)

(1) $\Delta s = \Delta l \left(1 + \frac{T}{EA}\right)$ (HOOKE'S LAW)
$= \Delta l \,(1 + \lambda T)$. $\lambda = \frac{1}{EA}$.

(2) $\Delta W = w\,\Delta l = \frac{w\,\Delta s}{1+\lambda T}$

(3) $W = T \sin\phi$.

(4) $d(T\sin\phi) = \frac{w}{1+\lambda T}\,ds$. Let $H = wc$
$H = T\cos\phi$.

(5) $T = H\sec\phi = wc\sec\phi$

(6) $d(wc\tan\phi) = \frac{w\,ds}{1+\lambda H\sec\phi}$; Let $\mu = \lambda H$.

$c\,d(\tan\phi) = \frac{ds}{1+\mu\sec\phi} = c\sec^2\phi\,d\phi$.

(7) $ds = c\,(\sec^2\phi + \mu\sec^3\phi)\,d\phi$ Note: $dx = ds(\cos\phi)$, $dy = ds(\sin\phi)$.

(A) $\frac{s}{c} = \tan\phi + \frac{1}{2}\mu(\sec\phi\tan\phi + gd^{-1}\phi)$.

(B) $\frac{x}{c} = gd^{-1}\phi + \mu\tan\phi$.

(C) $\frac{y}{c} = \sec\phi + \frac{1}{2}\mu\tan^2\phi$.

(8) $H = W\cot\phi = wl\cot\phi = wc$.

(D) $c = l\cot\phi$

(9) $\lambda W = \lambda H\tan\phi = \mu\tan\phi$. $W = \frac{1}{2}$ weight $= cw\tan\phi$

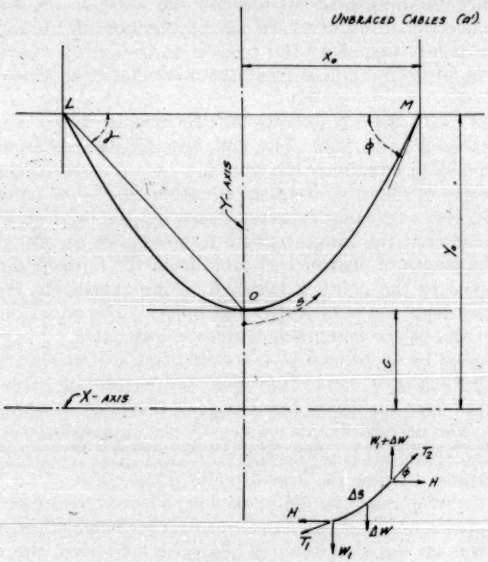
(B') $\frac{x}{c} = gd^{-1}\phi + \lambda W$.

(C') $\frac{y}{c} = \sec\phi + \frac{1}{2}\lambda W\tan\phi$.

(E) $T = H\sec\phi = W\csc\phi$.

Following the numbers of the equations given in the table it will be noted that equation (1) is an expression resulting from Hooke's Law applied to a differential length of the

FIG. 1.



Horizontal span.

curve. Equation (2) follows immediately since the weight of a section of cable before and after being stretched must be constant. Equation (3) is the expression resulting from the requirement that the slope of the tension at any point must coincide with the slope of the curve. The following equations are merely substitutions and integrations, resulting in equations (*A*), (*B*), and (*C*). The solution of these equations, eliminating the slope angle, is too complicated for this discussion and results in formulæ which are unwieldy. It is best to keep the tangent angle as a parameter. Tables for the trigonometric functions in these equations can be found

in any standard handbook, except that the expression gd^{-1}, the anti-gudermanian, is found only in advanced texts on hyperbolic functions. However, for approximate results, the graphs found below are sufficient.

Since the only external load on the cable is its weight, which acts vertically, the horizontal component of the tension at any point is equal to the tension at the point where the cable is horizontal and is therefore a constant for the entire cable.

The most useful equations for this case are those marked (B'), (C'), (D) and (E). The first two are derived from the corresponding equations (B) and (C) by the substitution of the weight of the cable from the equation (8). The equation (D) is a very important expression showing the relation which exists between the tangent angle to the curve at any point and the length of unstretched cable from the lowest point of the curve to the point considered. This expression results from the requirement of perfect flexibility. The equation (E) is the value of the maximum tension in any cable.

It must be noted that in this derivation of the equations, that the length of curve and that the weight of cable are measured from the lowest point and are therefore, for half the span. The vertical distance given by the expression y is not from the lowest point of the curve but from the X-axis which is a distance c below the lowest point of the curve.

In deriving the equation of the curve it was assumed that the supports are fixed, that the temperature is constant, that there was no initial tension in the cable and that the only load acting was the weight of the cable. The corrections involved in the equations for each one of the above items will now be considered.

Assuming that there is a change in temperature, 3 conditions will control.

(*a*) The weight of the cable does not change.

(*b*) The length of any portion of the cable increases by an amount equal to the coefficient of elongation for temperature on a given scale times the rise in temperature in degrees of that scale times the initial length. Of course if the temperature drops the increase will be negative.

(*c*) The length of span between fixed supports does not change.

These three conditions are given the corresponding numbers in Table (*b*). The rest of the derivation is purely an

TABLE (*b*)

UNBRACED CABLES (b).

HORIZONTAL SPAN.
TEMPERATURE CHANGE.

THREE CONDITIONS: (a) $\frac{\Delta W}{\Delta t} = 0$; (b) $\frac{\Delta x_o}{\Delta t} = 0$; (c) $\Delta l = \varepsilon l \Delta t$.

(1) $\frac{dW_o}{dt} = 0 = \frac{d}{dt}(w_o c_o \tan\phi_o)$

$= w_o c_o \sec^2\phi_o \frac{d\phi_o}{dt} + c_o \tan\phi_o \frac{dw_o}{dt} + w_o \tan\phi_o \frac{dc_o}{dt}$.

(2) $l_t = l_o(1+\varepsilon\Delta t)$

$w_t l_t = w_o l_o = w_t l_o (1+\varepsilon\Delta t)$.

$\Delta w = -w_o + w_t = -(\varepsilon\Delta t) w_t$.

$\frac{dw}{dt} = -\varepsilon w$.

(3) $\frac{dx_o}{dt} = 0 = \frac{d}{dt}(c_o \operatorname{gd}^{-1}\phi_o + c_o \lambda W_o)$

$= c_o \sec\phi_o \frac{d\phi_o}{dt} + (\operatorname{gd}^{-1}\phi_o + \lambda W)\frac{dc_o}{dt} + c_o\lambda \frac{dW_o}{dt}$.

(4) $\frac{d\phi_o}{dt}(c_o \sec^2\phi_o) + \frac{dc_o}{dt}(\tan\phi_o) = -\frac{dw_o}{dt}\left(\frac{c_o \tan\phi_o}{w_o}\right)$

$= +\varepsilon c_o \tan\phi_o = \varepsilon l_o$.

(5) $\frac{d\phi_o}{dt}(c_o \sec\phi_o) + \frac{dc_o}{dt}(\operatorname{gd}^{-1}\phi_o + \lambda W) = -c_o\lambda\frac{dW_o}{dt} = 0$.

(F) $\Delta c_o = \frac{\varepsilon l_o \Delta t_o}{\tan\phi_o - \frac{x_o}{c_o}\sec\phi_o} = \frac{\varepsilon c_o^2 \Delta t_o}{c_o - x_o \csc\phi_o}$.

(G) $\Delta\phi_o = \frac{\varepsilon \sin\phi_o \Delta t_o}{\sec\phi_o - l/x_o} = \frac{\varepsilon \sin\phi_o \Delta t_o}{\sec\phi_o - \frac{c_o}{x_o}\tan\phi_o}$.

$\phi_t = \phi_o + \Delta\phi_o$.

FOR y_t AND s_t USE FORMULAE (C') AND (A).

NOTE THAT $\mu_t = \lambda H_t = \lambda \mu_t c_t = \lambda \frac{(c_o + \Delta c_o) w_o}{1+\varepsilon\Delta t}$.

$H_t = w_t c_t = \frac{w_o(c_o+\Delta c_o)}{1+\varepsilon\Delta t} = \frac{H_o + w_o \Delta c_o}{1+\varepsilon\Delta t}$.

application of differential calculus and the resulting equations (*F*) and (*G*) give the increase in horizontal tension and in the slope angle for a small increase in temperature. To be perfectly accurate such increases should be infinitesimal in amount, but for the usual case the equations can be used in their form as given.

In such cases as counterweighted cables slung from high

flexible towers or from movable supports a change in span is possible. Under such change the conditions are: (*a*) that there is no change in the weight or the unstretched length of the cable, and (*b*) that the change in span with respect to itself is unity. Resulting from these two conditions the equations (*H*) and (*I*) are derived, which again give the variations in horizontal tension and slope angle with small change in span.

If the cable is strung with an initial tension so as to reduce the sag, the equations as derived above still apply, the value of the initial tension being added to that due to the weight. In using the formulæ it is necessary to adjust the value of c to suit such horizontal tension.

If in addition to its own weight a cable is loaded by a constant uniform load such as ice, the formulæ derived above apply and the new weight per unit length of the cable is the sum of the dead weight and the additional load.

If it is desired to keep a maximum constant tension in a cable the effect of temperature is different than if the cable is fixed at both ends. A constant tension in the cable can be arranged by stringing the proper unstretched length of cable between fixed supports. Change in tension and in slope angle due to changes in temperature are given in equations (*J*) and (*K*). (Table (*c*).)

The above equations cover practically all the possible cases for variation in curve and tensions in cables strung between horizontal supports. The use of these equations in various special problems and solutions by means of graphs is described below.

The expression gd^{-1} is known as the anti-gudermanian function. Its evaluation is by means of any of the following identities:

$$\begin{aligned}\mathrm{gd}^{-1}x &= \sin \mathrm{h}^{-1}(\tan x)\\ &= x + \frac{x^3}{6} + \frac{x^5}{24} + \frac{61x^7}{5040} + \ldots\\ &= \log_e \tan\left(\frac{3.1416}{4} \ldots + \frac{x}{2}\right).\end{aligned}$$

It must be noted that x is an angle measured in radians and that, in the last expression, the logarithm is to the natural

base. The first expression is readily solved with tables of the natural tangent and hyperbolic sine functions.

TABLE (c)

UNBRACED CABLES (c).

HORIZONTAL SPAN.
CHANGE IN SPAN

TWO CONDITIONS: (a) $\frac{\Delta W}{\Delta x} = \frac{\Delta l}{\Delta x} = 0$. (b) $\frac{\Delta x}{\Delta x} = 1$.

(1) $\frac{dl}{dx} = \frac{d}{dx}(c_o \tan\phi_o) = \tan\phi_o \frac{dc_o}{dx} + c_o \sec^2\phi_o \frac{d\phi_o}{dx} = 0$.

(2) $\frac{dx}{dx} = \frac{d}{dx}(c_o\, gd^{-1}\phi_o + c_o \lambda W) = (gd^{-1}\phi_o + \lambda W)\frac{dc_o}{dx} + c_o \sec\phi \frac{d\phi_o}{dx} = 1$.

(H) $\Delta\phi_o = \frac{\sin\phi_o\, \Delta x}{l_o - \lambda_o \sec\phi_o}$

(I) $\Delta c_o = \frac{\Delta x}{\frac{\lambda_o}{c_o} - \sin\phi_o} = \left(\frac{c}{x} + \frac{c^2}{x^2}\sin\phi + \frac{c^3}{x^3}\sin^2\phi \cdots\right)\Delta x$.

$\phi' = \phi_o + \Delta\phi_o$.
FOR y' AND s' USE FORMULAE (C') AND (A).
NOTE THAT $\mu' = \lambda H' = \lambda w_o c' = \lambda w_o (c_o + \Delta c_o)$.
$H' = w_o c' = w_o (c_o + \Delta c_o) = H_o + w_o \Delta c_o$.

CONSTANT MAXIMUM TENSION
TEMPERATURE CHANGE.

TWO CONDITIONS: (a) $\frac{\Delta T}{\Delta t} = 0$. (b) $\frac{\Delta x}{\Delta t} = 0$.

(J) $\Delta\phi_o = \frac{\varepsilon \cos\phi_o\, \Delta t}{\sin\phi_o - \frac{c_o}{\lambda_o}(1 + \lambda T \cos^2\phi_o)}$.

(K) $\Delta c_o = \frac{\varepsilon c_o (1 + \lambda T \cos^2\phi_o)\, \Delta t}{(1 + \lambda T \cos^2\phi_o) - \frac{\lambda_o}{c_o}\sin\phi_o}$.

$\phi'' = \phi_o + \Delta\phi_o$.
FOR y'' AND s'' USE FORMULAE (C') AND (A).
NOTE THAT $H'' = w_o c'' = w_o (c_o + \Delta c_o) = H_o + w_o \Delta c_o$.
$\mu = \lambda H''$.
NEW $l'' = c' \tan\phi'' = (c_o + \Delta c_o)\tan\phi''$.

B. THE UNSYMMETRICAL ELASTIC CATENARY.

When a cable is suspended from two fixed points not on the same horizontal line the resulting curve is not symmetrical. Since the reaction at the lower support is equal to the tension in the cable at that point and can be replaced by the continuation of the cable, the curve is really a portion of a symmetrical elastic catenary. Measuring from the lowest point, at which

the tangent is horizontal, the curve is really two halves of elastic catenary with semi-span of x_1 and x_2. (See Table (*d*).)

At the lowest point of the curve the tension is horizontal and the value of c is the same for the two portions of the curve on each side of the low point.

Referring to the figure on sheet (*d*) and substituting values from the formulæ derived for the symmetrical elastic catenary, the expressions (*L*), (*M*), (*N*), are derived. These expressions

TABLE (*d*)

UNBRACED CABLES (*d*).

SLOPED SPAN
(FIG. 2)

(1) $a = x_1 + x_2 = c(gd^{-1}\phi_1 + gd^{-1}\phi_2 + \mu \tan\phi_1 + \mu_0 \tan\phi_2)$.

(2) $b = y_2 - y_1 = c(\sec\phi_2 - \sec\phi_1 + \frac{1}{2}\mu \tan^2\phi_2 - \frac{1}{2}\mu \tan^2\phi_1)$.

(3) $L = c(\tan\phi_1 + \tan\phi_2)$.

(4) B = TOTAL WEIGHT $= wL = wc(\tan\phi_1 + \tan\phi_2)$.

$\mu = \lambda H = \lambda cw$.

(*L*) $\frac{a}{c} = gd^{-1}\phi_1 + gd^{-1}\phi_2 + \lambda B$.

(*M*) $\frac{b}{c} = \sec\phi_2 - \sec\phi_1 + \frac{\lambda B}{2}(\sec\phi_1 \sec\phi_2)(\sin(\phi_2 - \phi_1))$.

$= \sec\phi_2 - \sec\phi_1 + \frac{\lambda B}{2}(\tan\phi_2 - \tan\phi_1)$.

(*N*) $\frac{L}{c} = \tan\phi_1 + \tan\phi_2$.

TEMPERATURE SPAN-CHANGE.

(*O*) $$\Delta c = \frac{c\varepsilon L\,\Delta t\,(A_1 + A_2)}{(A_1 + A_2)L - a(A_1 \sec\phi_2 + A_2 \sec\phi_1) + b(\sec\phi_1 - \sec\phi_2)}.$$

$A_1 = \tan\phi_1 + \lambda B \sec\phi_1$.

$A_2 = \tan\phi_2 + \lambda B \sec\phi_2$.

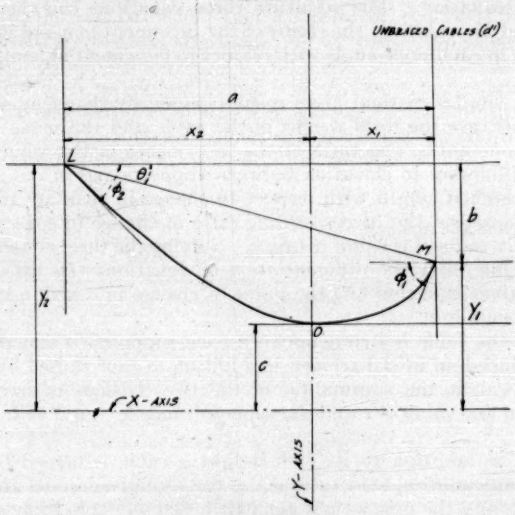
CHANGE IN SPAN.

(*P*) $$\Delta c = \frac{c^2(A_1 \sec\phi_2 + A_2 \sec\phi_1)\,\Delta a}{ca(A_1 \sec\phi_2 + A_2 \sec\phi_1) - Lc(A_1 + A_2) + b(\sec\phi_2 - \sec\phi_1)}$$

give the ratios between the total span, the difference in elevation between supports and the total unstretched length of cable to the value c in terms of the tangent angles at these supports measured from the horizontal and the total weight of the cable. In these three equations, for any definite case there are three knowns and three unknowns and a solution is therefore possible. It will be found that a solution by trial

is far easier than one by a rigid solution of the equations unless the values of the tangent angles are known.

FIG. 2.



Sloped span.

The variation in the horizontal tension with change in temperature is shown by equation (*O*). This equation is obtained by solving the three equations obtained by the differentiation of equations (*L*), (*M*) and (*N*) with respect to changes in temperature, keeping in mind that there are three variables. These three variables are the horizontal tension and the two slope angles. The total derivative with respect to the temperature, of the span and of the difference in elevation between supports is zero and of the total length of unstretched cable is the length of that cable times the coefficient of linear expansion. The detailed mathematics is not given in this discussion. In differentiating equations (*L*), (*M*) and (*N*), the total differential of the left hand side of the equations is equal to the summation of the partial differentials of the

right hand side of the equations. Before such differentiation is performed, each equation is multiplied by the value c and then the three variables, c and the two slope angles, appear in each equation. The resulting three equations can then be solved for values of the ratios of the increment in c and increment in each slope angle with respect to increment in temperature.

A similar method gives results caused by change in span. In this case the total weight of the cable and the value of b are constant. The ratio of the increments in the values of the difference in elevation between supports and of the total unstretched length with respect to change in span are therefore zero and the corresponding ratio of change in span with respect to itself is equal to unity. Solving the three equations resulting from the differentiation of equations (L), (M) and (N) gives equation (P) the value of change in c with respect to change in a.

If the cable is strung between fixed supports so that there is induced an initial tension in addition to that caused by its own weight the summation of the two tensions is used to obtain the value of c and the same formulæ are used with this new c.

If in addition to its own weight a cable is loaded by a constant uniform load such as ice, the formulæ derived above apply, and the new weight per unit length of cable is the sum of the dead weight and the additional load.

The above equations cover practically all the possible cases for variation in curve and tensions in cables strung between supports not at the same level. The use of these equations in various special problems is described below.

C. CONCENTRATED LOAD ON WEIGHTLESS STRING WITH HORIZONTAL SUPPORTS.

In order to study the effect of concentrations hung on cables of various types, two methods of attack are possible. If the cable is heavy with respect to the concentration, the sag and tension of the cable by itself should first be determined and to these values can be added the sag and tension resulting from a concentration placed upon a cable of the same length supported between the same supports. If the weight of the concentration is large compared to the weight of the

cable, the cable weight can be divided into four parts, the weight of each part as divided by the location of the concentration being distributed equally to the corresponding support and to the concentration. As a result the concentration is increased by one-half the weight of the cable, the other half being carried directly to the support. The cable is then studied as a weightless string with a single concentration. It is to be noted that in studying a cable with a moving concentration, the increased concentration is always the actual load plus half the total weight of the cable.

TABLE (e)

UNBRACED CABLES (e).

HORIZONTAL SPAN.
SINGLE LOAD
(FIG. 3).

$L = m+n.$

(Q) $z = \frac{l}{2a}\sqrt{(m+n+a)(m+n-a)(m-n+a)(n+a-m)}.$

(R) $(m+n)(m-n) = (f+d)(f-d).$

$n = \frac{L}{2} - \frac{a}{2L}(a-2d).$

$z = \frac{df}{a}(\tan\alpha + \tan\beta) = \frac{mn}{a}\sin(\alpha+\beta).$

EFFECT OF LOAD: P.

Δz = DEFLECTION DUE TO LOAD P.

(I) $\frac{P}{\sin(\alpha+\beta)} = \frac{T_m}{\cos\beta} = \frac{T_n}{\cos\alpha}.$

(S) $H = T_m\cos\alpha = \frac{P}{(\tan\alpha+\tan\beta)}.$

TOTAL INTERNAL WORK $= \frac{P^2[(\cos^2\beta)m + (\cos^2\alpha)n]}{2EA\sin^2(\alpha+\beta)}.$

DEFLECTION $= \frac{\partial W}{\partial P} = \frac{PL[m\cos^2\beta + n\cos^2\alpha]}{EA\sin^2(\alpha+\beta)}.$

(T) $= \Delta z = \frac{\mu P[m\cos^2\beta + n\cos^2\alpha]}{\sin^2(\alpha+\beta)}$

TEMPERATURE CHANGE.

(U) $\Delta z_t = \varepsilon\,\Delta t\,\frac{mn}{a}\sin(\alpha+\beta)$ (APPROXIMATE).

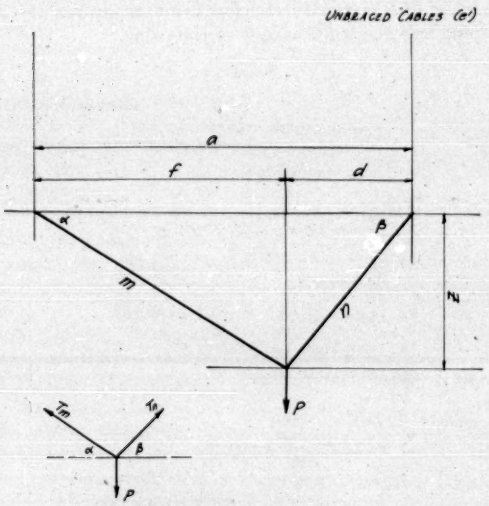
FOR $z = \frac{mn}{a}\sin(\alpha+\beta).$

(V) $\Delta H_t = \frac{-P\varepsilon\,\Delta t}{(1+\varepsilon\,\Delta t)(\tan\alpha+\tan\beta)}.$

In any case the equations derived below may be used. They cover all possible cases of a single concentrated load hung from a weightless string supported from either level or non-level points.

Taking the simplest case of a single concentration hung from level supports (see Table *e*), the geometry of the triangle

Fig. 3.



Horizontal span—single load.

so formed is given in equations (*Q*) and (*R*). The vertical altitude of the triangle is given in terms of the lengths of the three sides of the triangle and also in terms of the slope angles at the supports, the horizontal span, and the two portions of the span as determined by the location of the load.

Equation (*R*) also gives a relation between the length of the two portions of the string and the corresponding segments of the horizontal span.

The equation of equilibrium at the location of the con-

centrated load gives relations between the load, the tensions acting in each portion of the string and the two slope angles. The value of the horizontal component in both portions of the string is a constant of all points. Its value is given in equation

TABLE (f)

UNBRACED CABLES (f).

HORIZONTAL SPAN
SINGLE LOAD.

CHANGE IN SPAN.

$$(W) \quad \Delta z_a = \frac{z\,\Delta a}{[L\cos(\alpha+\beta)-a]}.$$

$$(X) \quad \Delta H_a = \frac{-P\,\Delta a}{(\tan\alpha+\tan\beta)^2}\left[\frac{\sec^2\beta}{n}+\frac{\sec^2\alpha}{m}\right].$$

CONSTANT COUNTERBALANCE.

FIXED END AT LEFT.
RIGHT END TO HAVE CONSTANT TENSION: T.

$$(1) \quad V_n = \frac{d}{a}P = T\sin\beta.$$

$$(Y) \quad \sin\beta = \frac{dP}{aT}. \qquad \tan\beta = \frac{1}{\sqrt{\left(\frac{aT}{dP}\right)^2-1}}.$$

$$(2) \quad z = f\tan\beta.$$

$$(Z) \quad \tan\alpha = \frac{f}{d}\tan\beta = \frac{fP}{\sqrt{(aT)^2-(dP)^2}}.$$

$$(3) \quad m+n = f\sec\alpha + d\sec\beta = L.$$

$$(T) \quad \Delta z = \frac{\mu P\,[m\cos^3\beta+n\cos^3\alpha]}{\sin^2(\alpha+\beta)}.$$

(*S*). It is to be noted that if additional loads are now added to the string the value of the horizontal tension will change. However, if all the loads are replaced by a single concentration acting at the center of gravity of the loads, the value of the horizontal tension can be determined in terms of those total loads and the two slope angles.

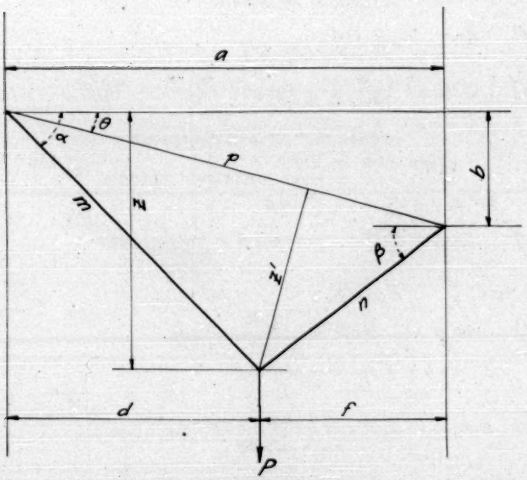
In other words, in any actual case in which the slope angles can be measured and in which all the loads including the weight of the cable itself are known, the horizontal component of the tension acting in the cable can be determined, and with the

slope angles known, the maximum tensions, occuring at the supports, can also be determined.

Returning to the first case, the effect of the concentration acting on the string which has elastic properties is to cause a deflection due to the stretching of the string. The amount

FIG. 4.

UNBRACED CABLES (ff)



Oblique span—single load.

of this deflection, given in equation (*T*), is determined by differentiating the expression for the total internal work in the system with respect to the value of the total load. The expression for the total internal work is given in equation (3). The total sag of a string with a concentration is equal to the geometric expression in equation (*Q*) plus the deflection value given in equation (*T*).

The effect of change in temperature, assuming that the horizontal span remains constant, is to increase the sag and

decrease the horizontal component of the tension. Once the new sag is determined, assuming that the location of the concentration does not move horizontally, the new values of the slope angles can easily be determined. Although it is not absolutely true that the string can continue in its triangular shape under such conditions, the error involved can not be very large. The increases in sag and in horizontal component of the tension are given in equations (U) and (V). It is to be noted that the former equation is approximate in that the horizontal movement of the load was disregarded. The equations are given for a positive increase in temperature. For a decrease in temperature the value for the increment of temperature must be substituted as a negative quantity in these equations.

To determine the effect of change in span, a similar process, in which it is assumed that the total length of string remains constant whereas the differential of span with respect to itself is unity, results in equations (W) and (X).

For the case where a string has a constant maximum tension at one end, because of a constant counterbalance arrangement, the values for the two slope angles in terms of the load, maximum tension and the geometric quantities fixing location of load are given in equations (Y) and (Z). The equations given above can then be used for determining all other quantities.

D. CONCENTRATED LOAD ON WEIGHTLESS CABLE WITH NON-HORIZONTAL SUPPORTS.

When a string is suspended from two fixed points not on the same horizontal line the effect of a single concentration is determined in the manner similar to the case of horizontal supports. Referring to Table (g), the value of the vertical distance between the higher support and the location of the load is given from the geometry of the resulting triangular in equation (Q'). If the slope angles at the two supports are measured from the horizontal the expression for the horizontal component of the tension, which is a constant at all points, is the same as in the case of a string hung between horizontal supports. The additional sag due to elastic developments is also the same.

The expressions for the effect of temperature which are given in equations (U') and (V') are exactly the same as the corresponding expressions (U) and (V).

TABLE (g)

UNBRACED CABLES (g).

OBLIQUE SPAN

SINGLE LOAD.

(FIG. 4.)

$$(Q') \quad z = \frac{bd}{a} + \frac{1}{2a}\sqrt{(m+n+p)(m+n-p)(m-n+p)(n+p-m)}$$
$$= d\tan\theta + z'\sec\theta$$
$$= \frac{d\sin\theta + z'}{\cos\theta}.$$

$$(S') \quad H = \frac{P}{\tan\alpha + \tan\beta}.$$

$$(T') \quad \Delta z = \mu P \frac{m\cos^2\beta + n\cos^2\alpha}{\sin^2(\alpha+\beta)}$$

TEMPERATURE CHANGE.

$$(U') \quad \Delta z_t = \Delta z_t' \sec\theta = \varepsilon\,\Delta t\,\frac{mn}{a}\sin(\alpha+\beta).$$

$$(V') \quad \Delta H_t = \frac{-P\varepsilon\,\Delta t}{(1+\varepsilon\Delta t)(\tan\alpha+\tan\beta)}.$$

CHANGE IN SPAN.

$$(W') \quad \Delta z_a = (z - d\tan\theta)\left(\frac{\sin^2\theta}{a} - \frac{1}{(m+n)\cos(\alpha+\beta) - a\sec\theta}\right)\Delta a$$

$$(X') \quad \Delta H_a = -P\,\Delta z_a\left[\frac{\frac{\tan\alpha}{m} + \frac{\tan\beta}{n}}{(\tan\alpha+\tan\beta)^2}\right]$$

CONSTANT COUNTERBALANCE

$$(Y') \quad \sin\beta = \frac{dP}{aT}.$$

$$(Z') \quad \tan\alpha = \frac{b}{d} + \frac{f}{d}\tan\beta.$$

A change in the horizontal span, however, gives more complicated expressions for the corresponding changes in sag and horizontal component of the tension. These are given in equations (W') and (X').

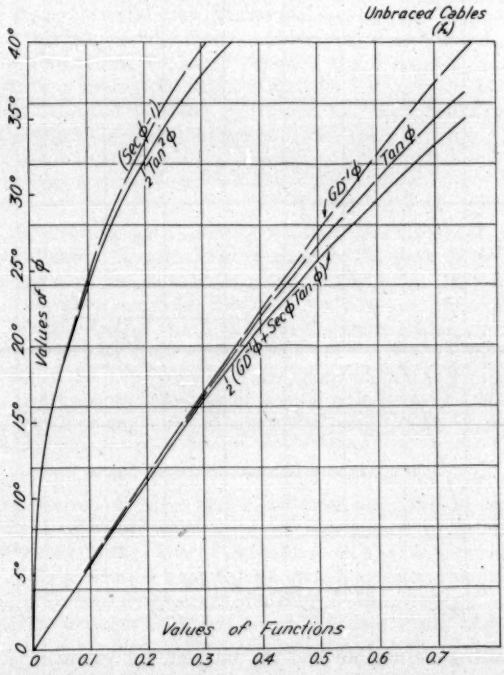
For the case of constant counterbalance the values of the two slope angles at the supports, measured from the horizontal are given in equations (Y') and (Z').

E. USE OF DIAGRAMS TO SOLVE EQUATIONS AND PROBLEMS.

Equations derived in this paper are for such large installations that the elastic stretch of the wire or cable is appreciable. For such conditions diagrams or graphs can not be developed to cover the great number of possible cases.

It will be noted that all the above terms for the elastic catenary, if the terms containing the expression u are elimin-

FIG. 5

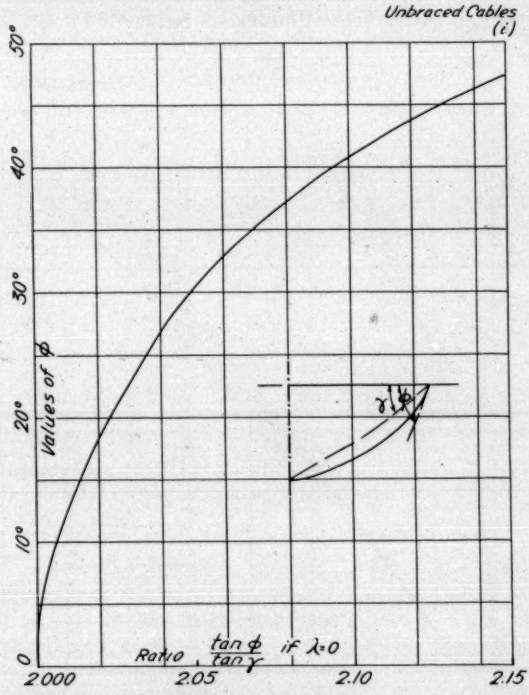


ated, that is to say the elastic stretch is zero, all become expressions of the ordinary catenary. For short spans and for spans

with a small sag, the catenary equation is a very close approximation to the final results.

The curves shown in Fig. 5 are to be used for approximate solutions only. They show the values of the secant,

FIG. 6



the anti-gudermanian and the tangent, for values of angles from zero to 35°. For more accurate work, the value of the anti-gudermanian should be computed either by means of tables or the expansion series.

The usual procedure for the solution of these equations is a method of trial, assuming values of one of the unknown variables and checking back to determine the accuracy of the assumption. In this type of procedure it is quite useful to compute three or four values of the unknown quantities based upon assumed values of one of the unknowns and plot graphs of the resulting quantities. From these graphs the correct values of the unknowns can be chosen.

The expression "sag ratio," usually defined as the ratio of the sag to the total span, has no real meaning in the general case. This is especially true of cables with sloped chords.

The tangent of the angle between the horizontal and a straight line joining the support and the lowest point of the cable is the ratio between $y - c$ and the corresponding x. Substituting from the equations in Table (a):

$$\tan \gamma = \frac{y - c}{x} = \frac{\sec \phi + \frac{1}{2}\lambda W \tan \phi - 1}{g d^{-1}\phi + \lambda W}.$$

If the elastic stretch is disregarded and the curve becomes a pure catenary the ratio of the tangents of the slope angle and chord angle at the support is given in the graph in Fig. 6. If the curve is a parabola, this ratio is a constant and equals 2.0. It will be noted that the larger the slope angle the more the curve deviates from a parabola.

For any definite case where the chord angle can be measured, and knowing the weight and elastic property of the cable, the slope angle can be computed from the above equation.

F. EFFECT OF WIND ON VARIOUS CONDITIONS.

The loading of cables acts in all cases in a vertical plane. The effect of wind, which pressure is assumed to act horizontally is to tip the plane of the cable from a vertical position. If an oblique wind is assumed, the wind pressure can be resolved into two components, horizontal and vertical. The vertical component is added to the vertical loads and the horizontal component is then considered as the effective wind pressure.

The amount of wind pressure is discussed below. The worst effect of wind pressure will usually occur when the wire

or cable is coated with ice since such coating increases the exposed area and the pressure of wind is a direct function of the exposed area.

Under any assumed or known conditions of vertical and horizontal loads, a cable can be designed to resist such loads without over-stress. The resultant of horizontal and vertical loads is an oblique load which must determine the plane of the cable. Stresses are then figured as if the oblique plane were vertical with the resultant load as the weight or forces acting on the cable. The same formulæ derived above for the vertical plane then apply. This is even true for the case of counter-balanced cables if, as is usually the case, the sheave at the top of the tower over which the cable runs to a counter-weight is designed properly, and can take the position of the plane of the cable.

G. SPECIAL PROBLEMS.

In any special problems certain quantities are either known or must be assumed and other quantities are to be found or determined. In the case of fixed-end cables such as antenna wires, suspension bridge wires, transmission lines, etc., the span and difference in elevation of supports as well as the elastic property and unit weight of the wire are known. The remaining unknowns are: the sag, unstretched length of wire, maximum tension, horizontal tension, slope angle, and chord angle. For an existing condition some of these unknowns can be measured and the rest determined; for a design, some of these unknowns must be assumed before the rest can be computed.

The same applies to counterweighted cables such as antennas or transmission lines designed with counterweighted towers except that the maximum tension in the wire at the counterweight is always known, eliminating one of the unknowns of the previous case. In both of these cases maximum stress conditions will be found under full wind and ice loads.

For the case of guys this method of computation need only be used for wire rope guys attached to very high towers. The use of guyed towers for radio purposes is increasing and the computation of the stresses in the guys under various wind loads as well as under various deformations of the mast require considerable precision for accuracy. In general the

design must first be made upon assumed sections of both the mast and guys and stresses checked in accordance with the formulæ for sloped cables. In applying the equations for cables to the case of guys it will usually be found that both ends of the guy are on the same side of the catenary curve with reference to the point of horizontal tension. The equations still apply but it must be noted that the sign of the lower tangent angle is negative since it is measured up from the horizontal.

The problem of designing cableways for carrying loads is more difficult. Where the load is a heavy single concentration such as occurs in car-ferries, freight or passenger conveying long span cables, the end reactions for any position of the load can be determined. The cable can then be designed or investigated without load and also investigated as a weightless cable with a single concentration, taking the weight of the cable into account by adding half the total weight to the concentration. The location of the load giving maximum stress in the cable is determined by trial. The formulæ for concentrated load on steel cable, taking into account the elastic stretch and temperature changes will give a sufficient number of equations for the solution of any one problem.

H. TYPE OF CABLES AND ELASTIC PROPERTIES.

The various types of wire ropes are described in detail in several available text books and wire rope catalogues. A number of these references are given in the bibliography. The elastic properties of wire rope vary with use. In general, if the value of the modulus of elasticity of a wire is one, the modulus of elasticity of a strand consisting of this type of wire is 0.6, the modulus of elasticity of a rope consisting of strand is 0.36 and the modulus of elasticity of a cable consisting of ropes is 0.216. In general, the twist due to a formation of a strand from a wire or a rope from strands or a cable from ropes reduce the modulus of elasticity to 0.6 of its former value. C. Bach found experimentally that the modulus of elasticity for wire rope is 0.375 $\times$ the modulus elasticity of the individual wire. Julius Divis as a result of a great number of experiments with twice stranded rope found the value of the modulus of elasticity as 0.36 $\times$ the modulus of elasticity of the individual wire.

Boomsliter in testing 6 × 19 cast steel hemp core hoisting rope, $\frac{3}{8}$ in. $\frac{1}{2}$ in., $\frac{3}{4}$ in., 1 in. and $1\frac{1}{4}$ in. sizes, found the modulus of elasticity of the new rope approximately 12,000,000 and after considerable use the value to be closer to 20,000,000 lbs. per. sq. in.

Umanski in experimental work with $1\frac{1}{16}$ in. wire rope tested a length of 55 ft. and determined an elastic modulus of 8,360,000. This was probably for a new cable.

Meales reports values of elastic modulus for various cables as follows: 17,000,000 lbs. per sq. in. for new cable and 20,000,000 for old cable, the types tested being 7 × 7, 7 × 19, and 7 × 37, wire strands weighing from 3.70 to 15.5 lbs. per ft. The pre-formed wire cable, known as true-lay, showed elastic modulus of 18,000,000 for 37 wire strand and 20,000,000 for the 19 wire strand. The suspender ropes used for the Delaware River Bridge were made up of the three operation strand, $2\frac{1}{4}$ in. diameter, 6 × 37, all wires laid in the same direction with a 7 × 7 independent wire rope center of Lang lay. This rope showed an elastic modulus of 8,000,000 on the first load test increasing to 12,00,000 after repeated loading.

The Copperweld Co. in its table of standards gives the following values for the elastic modulus:

20,000,000 lbs. per sq. in. for single wire varying from 0000 to 14 gauge, corresponding to 0.1662 to 0.00322 sq. in. respectively.

16,100,000 for 7 wire strands consisting of No. 5 wire.

19,900,000 for 7 wire strands consisting of No. 14 wire.

16,000,000 for solid copper wire.

26,000,000 for E. B. B. wire.

The coefficient of linear expansion due to changes in temperature for cables is less than the coefficient for the constituent wires. The Copperweld Co. gives the following values for the coefficient of linear expansion:

For Copperweld wire .0000072 per degree Fahrenheit.

For copper wire .0000096.

For E. B. B. wire .0000067.

The weight of ice which forms on wires or cables seems to be approximately constant for all wires of 1 in. or less in size. The weight is approximately that of an ice coating $\frac{1}{2}$ in. thick. Measurements made by Ryan of the weight of ice on

Michigan telephone wires shows a weight of 45 to 70 lbs. of ice per 100 ft. length of iron wire No. 13 and approximately the same weight on insulated telephone cable of 0.14 in. diameter.

The amount of wind load acting on wire ropes and cables depends to a large extent on the location. A great variation exists in the assumed loads. The so-called "heavy load" recommended by the U. S. Bureau of Standards is a ½ in. ice radial coating and 8 lbs. per sq. ft. exposed surface (including the ice) for wind. In designing radio antennas, Buel uses 80 lbs. per sq. ft. of exposed area for tropical locations.

The actual pressure of wind on wire rope has been tested by a number of experimentors and its value in lbs. per sq. ft. of projected area is given very closely by the formula $p = .0025\ v^2$, where v is given in miles per hour actual velocity.

This value is lower than the formula for pressure for wind on flat surfaces, but experiments show it to be quite accurate for a great variety for wires and ropes.

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NOTATION.

Uniformly loaded cables.

a: horizontal span between supports.
b: vertical difference in elevation between supports.
c: ratio of horizontal component in cable to weight of unit length of unstretched cable.
c_0: value of c at standard temperature and definite location of supports.
c_t: value of c at temperature t.
c': value of c after small change in span.
c'': value of c after small change in temperature with constant maximum cable tension.
Δc: increment in value of c.
l: unstretched length of cable from lowest point to point whose coördinates are x, y.
l_0: value of l at standard temperature.
l_t: value of l at standard temperature t.
l': value of l after small change in span.
l'': value of l after small change in temperature with constant maximum cable tension.
s: length of actual curve for any case.
t: temperature in degrees.
μ: product of horizontal component of cable tension and elastic coefficient, λ, also equals the stretch per unit length of horizontal portion of cable.
w: weight of unit length of unstretched cable
w_0: value of w at standard temperature.
w_t: value of w at temperature t.
x and y: coördinates of curve.
x is measured from lowest point of curve.
y is measured from horizontal axis a distance of c below lowest point of curve.
A: cross section area of unstretched cable.
B: total weight of cable.
E: Young's modulus of elasticity.
H: horizontal component of cable tension.
L: total length of cable in unstretched condition.
T: tension in cable at any point.
W: weight of cable from lowest point to point whose coördinates are x, y.

θ: angle of deviation from horizontal of line joining supports.
ϵ: coefficient of linear expansion per degree temperature.
λ: coefficient of linear strain in tension.
ϕ: angle of deviation of tangent to curve.
ϕ_1 and ϕ_2: end values of ϕ for non-symmetrical case.
$\Delta\phi$: increment to value of this angle.

Additional Notation for Concentrated Loads.

d and f: parts of horizontal span as divided by location of load.
m and n: corresponding parts of cable.
z: vertical position of load below higher support before elastic cable stretch.
Δz: increase in z due to stretch of cable.
z': normal distance between concentrated load and line joining supports.
P: concentrated load.
T_m and T_n: tensions in two parts of cable.
V_m and V_n: vertical reaction at two supports.
α and β: slope angles below horizontal at the two cable supports.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

NEW BASIS FOR ELECTRICAL UNITS.

THE minutes of the 1929 session of the International Committee of Weights and Measures, which have just been issued, record a definite step toward the adoption of electrical units consistent with those used in mechanics, and the establishment of a permanent means of coördination between the laboratories which maintain the units in different countries. The Committee also decided to take up the question of a uniform international unit of light.

On the subject of electrical units and standards, the International Committee had before it an extensive report prepared under the direction of an Advisory Committee on Electricity. This international Advisory Committee consisted of ten members, including representatives of six national standardizing laboratories. It was appointed in 1928, and, through national advisory committees and other organizations, secured practically a world-wide consensus of opinions on this subject. Its conclusions were in full accord with recommendations made by the American committee, and the report was prepared by the representative of the Bureau of Standards, Dr. George K. Burgess.

In accordance with the recommendations of the Advisory Committee, the International Committee at its regular biennial session held at Paris in June, 1929, adopted resolutions translated as follows:

A. With regard to the system of units.

1. Considering the great importance of unifying the systems of electrical measurements on a basis free from all arbitrary characteristics, the absolute system, derived from the c.g.s. system ought to be substituted for the international units for all scientific and industrial measurements.

* Communicated by the Director.

2. Since it is not possible at the present time to fix with the exactness which is desirable, and of which they are susceptible, the ratios between the absolute units derived from the c.g.s. system and the international units of current, of electromotive force, and of resistance, as these were defined by the International Congress at Chicago in 1893 and the London Conference in 1908, the Committee expresses the wish that researches be pursued to this end in the laboratories which are suitably equipped, following a program previously studied in accord with the Advisory Committee on Electricity.

B. The International Bureau of Weights and Measures will establish:

1. An organization to arrange for systematic interchange of standards and to assure the compilation of the results of comparisons made by the national laboratories;

2. A laboratory to which material standards representing the results obtained in different countries can be brought for precise comparisons;

3. A depository for reference standards and working standards, including standards of inductance and capacitance, with the apparatus necessary for the comparison of other standards with those of the Bureau.

C. The International Committee on Weights and Measures, under the authority given it by the General Conference, will have the responsibility of deciding and of promulgating the values to be employed for the practical standards, and will determine the date of a new revision. To this end, and for the arrangement of experimental determinations and the analyses of their results, the Advisory Committee will continue to give its counsel to the International Committee in the exercise of the functions which the General Conference has delegated to the latter Committee. The International Committee on Weights and Measures will draw up by-laws to govern the organization and functions of the Advisory Committee.

D. The International Committee on Weights and Measures will take up with the proper authorities such

measures as are necessary to obtain special facilities for the passage across frontiers of packages containing standards, whether accompanied or not.

The by-laws mentioned under paragraph C were also provisionally adopted, but are not to be published until the Advisory Committee meets again.

Refering to the lack of concordance between the candle-power standards of different countries, the Committee adopted the following resolutions:

1. The International Committee on Weights and Measures, considering the importance of unifying the methods used in photometry, decides to study the question of the adoption of an international system of units of light.

2. For this purpose, the International Committee charges the Advisory Committee on Electricity to advise it on all questions relating to the methods of measurement and to the units and standards of light.

3. The International Committee on Weights and Measures authorizes the Advisory Committee to solicit for the study of these questions, the collaboration of the International Commission on Illumination.

It was also decided that the Advisory Committee would be asked to meet again in about a year to consider the question of photometry and the installation of electrical laboratories at the International Bureau of Weights and Measures.

DIRECTIVE RADIO BEACON SYSTEM.

WITH the aural type directive radiobeacon system now in use on the airways of the United States, it is the practice to operate the radiobeacon and radiotelephone weather broadcast stations on the same radio frequency, interrupting one service when the other is in use. This permits the pilot to use these services without the necessity of retuning the radio receiving set from the beacon to the phone service or vice versa. The two services are, nowever, not available simultaneously.

It would be advantgeous if the pilot could have available

his course indication even while listening to the weather broadcast. The visual type of radiobeacon offers possibilities for the simultaneous transmission of the two services on the same radio frequency. The modulation frequencies used with the visual beacon are 65, 86.7 and 108.3 cycles, respectively. Intelligible speech does not require modulation frequencies below 300 cycles. The two sets of modulation frequencies may therefore be transmitted on the same carrier frequency. On the airplane a receiving set tuned to the transmitted carrier frequency may be employed, and a suitable filter circuit arrangement provided in the receiving set output so that the frequencies above 300 cycles are supplied to the head telephones, and those below 300 cycles to the vibrating reed course indicator. The bureau is making a study of the most practicable transmitting circuit arrangements for the simultaneous transmission of the radiobeacon and radiotelephone signals. Preliminary experiments to test the practicability of these arrangements have been started.

VIBRATING REED COURSE INDICATOR.

THE vibrating reed course indicator (used with the visual type radiobeacon) has heretofore been mounted on airplane instrument boards at least 18 inches from the magnetic compass, so as to avoid any effect upon the compass readings.

Recent experiments at the bureau have shown that such a separation is unnecessary if sheet iron is used for the external mounting of the reed indicator. Preliminary measurements show that the indicator may then be located within 3 inches of the magnetic compass without affecting the compass readings.

EFFECT OF ULTRA-VIOLET RADIATION UPON THE TRANSMISSION OF SPECIAL WINDOW GLASSES.

FOR the past two years the Bureau of Standards has been making an extensive study of the effect of ultra-violet light upon the transparency of window glasses and glass substitutes to be used for transmitting short wave-length ultra-violet radiation (1) from artificial sources and (2) from the sun. Part of the information thus obtained is given in *Bureau of Standards Journal of Research*, Vol. 3, No. 5, p. 629; November, 1929. The reprints of this article, Research Paper No.

113, are obtainable from the Superintendent of Documents, Government Printing Office, Washington, D. C.

One of the high points recorded in this paper is that some of these glasses and glass substitutes undergo a photochemical change, with a marked decrease in the transmission of ultraviolet radiation, which renders them unsuitable for filters of artificial sources of radiation.

Another point of interest mentioned in that paper is that the unfiltered radiation from artificial sources seemed to produce a somewhat greater decrease in transmission than sunlight, but that sufficient time (one year) had not elapsed to determine whether the glasses which were exposed to the sun had attained a minimum transmission.

From a further study of these various makes of glasses which have now been exposed to the sun and weather continuously for 1½ to 2 years (some samples were exposed through two winters and two summers) it appears that, within the experimental errors of observation and handling such material, there is no appreciable recovery or increase in transmission in winter above the minimum value attained in summer; also that it requires an exposure of 3 to 4 months to the hot summer sun and weather in order to make certain that solarization is fairly complete, especially in certain makes of glass that solarize but slowly.

After this initial solarization, an exposure of these glasses to the mercury arc lamp produces a further decrease in transmission, varing from 1 to 8 per cent. at 302 millimicrons, depending upon the sample tested. The difference in the decrease in transmission produced by these two methods of so-called photochemical stabilization is small, averaging less than 6 per cent. for the earlier productions of Helioglass, Holviglass, Sunlit, and Vitaglass, and it does not modify the rating of the relative performance of these glasses (see Table 2, of Research Paper No. 113) in which the public is interested. For example, the average transmission of the samples of Helioglass described in Table 2 of Research Paper No. 113, after stabilization by exposure to the sun would be expected to be 46 per cent. as compared with 40 per cent. under the mercury arc.

As mentioned on page 652 of Research Paper No. 113, the stabilization tests under the mercury lamp indicated that the

newer productions of Helioglass and Vitaglass showed an improvement in transparency. This has now been verified by exposure of these glasses to the sun for 10 months. For example, the sample of Vitaglass, which transmitted 62 per cent. when new, decreased in transmission to 43 per cent. after solarization for 10 months, and to 30 per cent. after exposure to the mercury lamp. Similarly, a recent production of Helioglass transmitted 68 per cent. when new, 59 per cent. after solarization for 10 months, and 46 per cent. after exposure to the mercury arc—a difference of about 13 per cent. produced by these two methods of depreciation. It is interesting to note that the relative performance of these glasses remains the same.

Recently it was found by Wood and Leathwood (*Nature*, Sept. 21, 1929) and by the Bureau of Standards that exposure of the glass to the sun *after exposure* to the mercury arc lamp *increases the transmission* in the ultra-violet to closely the minimum value that was obtained by exposure only to the sun. In fact, in the Bureau of Standards' tests, which were made on glasses that had been exposed to the sun for a long time before exposure to the mercury arc, it was found that, on further exposure of these glasses to the sun, after exposure to the lamp, the transmission increased to a higher value than the minimum attained by exposure only to the sun. This overshooting in transmission, which ranges from 1 per cent. to about 6 per cent. above the minimum attained by exposure only to sunlight, occurred in 17 out of 20 samples of different makes of glass, including Helioglass, Holviglass, Neuglas, Ultra-Violet Glass, Ultravit, and Vitaglass. Whether this condition remains permanent will require measurements on these glasses after exposure to the sun, especially during the summer.

This recovery in transmission appears to be selective to certain wave-lengths of the exciting light. For example, one sample of glass was exposed to the mercury arc and then cut into nine pieces, one of which was exposed directly to the sun. The others were exposed under black paper (opaque to all rays) and under filters which transmitted bands of infra-red, red, yellow, green, blue, violet, and ultra-violet rays of the sun. The recovery in transmission was practically the same under all these filters except the ultra-violet, Corex G986A, which

produced the greatest recovery, and Corning pale blue green, G584, which produced the second greatest recovery. From this it appears that the spectral region of 365 millimicrons produces the greatest recovery in transmission.

It is to be noted that all these depreciation tests on different kinds of glass were made in duplicate, by exposure (1) to the mercury arc lamp (and other artificial light sources) and (2) to the sun, and that in all makes of glass (except Corex A) a close similarity was found in the depreciation in transmission produced by these two sources of ultra-violet radiation. That is to say, glasses (for example, Corex D), which undergo little or no change in transmission on exposure to the mercury arc, undergo no appreciable change in transmission on exposure to the sun; and glasses, also different melts of the same kind of glass, which show the greatest decrease in in transmission on exposure to the mercury arc, show the greatest decrease in transmission on exposure to the sun. All deductions regarding the depreciation in transmission of different makes of glass, and different melts of the same kind of glass, are based upon experiments in sunlight, supplemented by experiments with artificial sources of radiation, leaving no doubts regarding their performance.

It seems impracticable to spend several (preferably summer) months in order to obtain the performance of every melt of glass by exposure to the sun. The depreciation in transmission by exposure to the mercury arc appears to be a little more severe than sunlight. But in view of the fact that differences of 10 to 22 per cent. in transmission are found in new samples of certain kinds of glass, selected at random from the same shipment (and presumably from the same melt), the mercury arc appears to be a ready means for determining, in a few (3 to 5) hours, the so-called stabilized transmissive properties of a batch of glass. From these data, and from the above-mentioned corrections, the transmission that it will probably attain after solarization by prolonged exposure to the sun can be predicted.

ACCELERATED WEATHERING OF PAINTS.

A comparison of outdoor and accelerated (combined light-spray only) exposures of the same white paints is giving some

interesting information. The paints under test are white lead, lead-zinc, and lithopone. Thus far results have been obtained on six weeks of exposure in the accelerated cycle, taking a set of panels out at the end of every week. The white lead paints are failing by chalking, fine checking, and a gradual disappearance of the paint coat (by chalking off). The lead-zinc paints are failing by cracking, resulting in a flaking or crumbling of the paint coat. The lithopone paints chalk heavily, crack, and soon disappear from the wood. White lead shows up the best, lead-zinc next, and lithopone a poor third. Lithopone paint fails in about three weeks, and lead-zinc in about five weeks. The white lead paints have not reached (after 6 weeks exposure) as bad a condition as the lithopone paints at the end of three weeks. A comparison, on a quantitative basis, of the relative merits of these paints is difficult, because of the different types of failure for the various paints. Using three weeks as the period of failure for the lithopone paints, the other paints appear to be in somewhat the following ratio at the present time:

Lithophone Paint.	Lead-Zinc Paint.	White Lead Paint.
3	5	More than 6

PERMANENCE OF PAPERS.

IN response to urgent demand from a number of sources—paper manufacturers, paper dealers, librarians, publishers, scientific organizations and the public in general—the Bureau of Standards instituted a research to find more definite information on the permanence qualities of papers used for written and printed records, and on the preservation of such records.

The program adopted was made up of: (1) tests of the current commercial rag fiber and wood fiber products including the fibrous raw materials; (2) tests of similar papers made in the bureau's paper mill and therefore having a definitely known history; (3) inspections and tests of papers of known age; (4) a study of means of overcoming influences found to be harmful to the life of papers, and (5) research to find the nature of the reaction of paper celluloses to deteriorating influences.

Some work has been done on item No. 1, and a progress report "A Study of Purified Wood Fibers as a Paper Making Material" published in the *Bureau of Standards Journal of*

Research, Vol. 3, No. 3, p. 469; September, 1929 (Research Paper No. 107).

Work is now in progress on item No. 2 of the program. The types of papers to be made are, (1) sulphite-soda book, (2) sulphite bond, (3) bond made from purified wood fibers, (4) five grades of rag writing and bond papers using all the various grades of rags. The objectives of this work are, (1) to make the best papers possible from the various fibrous materials so as to obtain a more exact comparison of the paper-making quality of the raw materials than is possible with commercial papers, (2) to introduce variables suscepted of being harmful, such as overbleaching, high acidity, and overbeating, for the purpose of observing their effects on the different fibrous materials.

Intensive study of items (3) and (4) has been made possible by a fund granted by the Carnegie Foundation, which is particularly interested in the preservation of publications. This fund is being administered by the National Research Council. Surveys of public libraries to correlate deteriorative effects with storage conditions are in progress, and laboratory studies of the deteriorating effects indicated by the library surveys are being initiated.

The obvious point of attack on item (5) is to find more definitely the nature of the products formed by degradation of cellulose. With this information, a study of the mechanism of the degradation will be the next logical step. A knowledge of these facts will enable the bureau (*a*) to inhibit or delay degradation or (*b*) to predict whether or not degradation will occur. Fundamental physical and chemical studies of cellulose are being initiated for this phase of the work

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WHITE STAINLESS MINERAL OIL LUBRICANT FOR TEXTILE KNITTING MACHINES.

The hosiery and underwear industries have had considerable difficulty with stains on their fabrics produced by the lubricating oils used on knitting machines. Lubricating oils are used on all movable parts of the machines, various parts of which come in contact with the yarns. It is therefore not practicable to keep the oil off the yarn. Because of the conditions of manufacture, the knitted fabric is stored in bins or

boxes before dyeing and finishing. This storage period may cover six or more months during which time oil stains develop through oxidation of the oil. These stains are very difficult if not impossible to remove in subsequent finishing processes.

In the hope of obtaining a stainless lubricating oil, a study was undertaken over two years ago by the National Association of Hosiery and Underwear Manufacturers coöperating with the Bureau of Standards through its Research Associate, E. M. Schenke. The oil refineries were asked to coöperate and 12 of them submitted 36 different lubricating oils. All of these were submitted to the following tests:

1. Exposure on knitted fabric to daylight for six months.
2. Exposure on knitted fabric to carbon arc lamp light (Fade-Ometer), for 18 hours.
3. Viscosity.
4. Flash point.
5. Corrosion.
6. Neutralization number.
7. Lubricant value.
8. Presence of unsaturates.

These tests have yielded the following tentative specifications:

WHITE STAINLESS MINERAL OIL LUBRICANT FOR ALL KNITTING MACHINES.

a. Viscosity at 100° F.—Not less than 75 nor more than 100.

b. Flash point—Not less than 300° F.

c. Color—Must pass the latest edition pharmacopœia specification.

d. Corrosion test—A clean copper strip shall not be discolored when submerged in the oil for 3 hours at 212° F.

e. The neutralization number shall not exceed 0.10.

f. Acid test—Pour 10 cc. of the oil into a test tube. Add 5 cc. C.P. sulphuric acid, 1.84 specific gravity. The liquids shall remain colorless.

Three of the submitted oils, each from a different refiner, fulfilled the requirements of the tentative specification.

The specification has been submitted to the refiners for comment.

VAPOR PRESSURE AND VAPOR LOCK.

A PAPER on gasoline vapor pressures, presented at the annual meeting of the American Petroleum Institute in Chicago on December 4, was introduced as follows by Dr. H. C. Dickinson of this bureau.

At least two things are required of every motor vehicle engine: it must start, and it must keep on running without too many interruptions. The ability to do these two essential things depends partly on the fuel with which it is supplied.

At low temperature, starting is difficult and gasoline is needed which has an adequate vapor pressure. On the other hand, when the engine and the carburetor float bowl are hot, the same gasoline which affords easy starting may fail, causing vapor lock and stopping of the engine.

Dr. Bridgeman and his staff have shown in previous papers that the temperature at which a start can be had may be predicted from the temperature at which 10 per cent. of the gasoline is evaporated in the standard distillation test, provided certain constants of the engine are also known.

In this paper he shows that this same point, viz., that at which 10 per cent. of the particular sample of gasoline is evaporated, is a measure of the liquid temperature at which this gasoline starts to boil. If the fuel boils in the carburetor, vapor lock and engine stoppage may be expected. If it never approaches the boiling point, no such trouble is likely to occur. Between the two there is a range of conditions such that a well designed carburetor and fuel system may avoid trouble while a poor one may cause vapor lock with the same fuel at the same temperature.

The value of this paper and others of the series to the engineer and designer of motor vehicles and to the oil refiner is this: The engine must start readily, for instance, at 0° F. To do so will require a fuel with 10 per cent. evaporated at 140° F. This fuel will boil if heated above 140° F. If the gasoline in the fuel lines on said engine ever approaches 140° F., vapor lock may occur. Therefore, there are two alternatives: (1) The engineer must so design the fuel lines,

pump or vacuum tank, and carburetor that vapor lock will not occur with this fuel, which probably means that the fuel must never reach the 10 per cent. point temperature in service; or (2) the user of the engine must have a different grade of gasoline for certain operating conditions.

The present paper offers an accurate means of comparing different gasolines as to their tendency to boil either in automobile fuel systems or in airplane fuel systems at any altitude. Work in progress aims to apply this information in predicting the minimum temperature at which vapor lock may occur. In the meantime, the exact temperatures in the fuel lines at which trouble does occur with any given automobile or airplane engine must be found by trial, using fuels of known 10 per cent. point.

NOTES FROM U. S. BUREAU OF MINES.*

SUBSTITUTION OF MANUFACTURED GAS FOR NATURAL GAS.

INDICATIONS are that a volume of city gas 100 per cent. greater than is now being used will be manufactured in the near future, according to the United States Bureau of Mines, Department of Commerce, which has conducted an economic study of the situation. Natural gas is being supplied in increasing amounts to a growing number of communities and industrial centers throughout the United States, in some instances being delivered through transmission lines several hundred miles long, says William W. Odell, in a report on the subject. During the seven years from 1922 to 1928 the number of domestic consumers increased from 3,015,000 to 4,366,000.

Much of the increased production of natural gas in the past few years is in the Southwest and on the Pacific coast. A market for gas is inevitably being created which can not be filled indefinitely by the natural product. Much of the demand thus fostered apparently must be satisfied in the future by manufactured gas, but obviously some of the industrial consumers will not use city gas as fuel. As the natural gas becomes exhausted manufactured gas must be generated in increasing amounts to maintain a supply equal to consumers' demands.

The raw materials from which substitute gas can be made comprise solid fuels, including coke and various kinds of coal, and petroleum or its products. For various economic reasons, the raw material from which the substitute gas of the future will be made is solid fuel. Preferably it should be a non-processed fuel, such as bituminous coal, unless it develops that processing yields by-products of such a high value that coke can be sold at a price comparable with coal. However, in this instance it can be used only for the production of water gas, producer gas, or similar gas.

* Published by permission of the Director, United States Bureau of Mines.

In general, manufactured city gas comprises as combustible constituents, chiefly and by volume, hydrogen and hydrocarbons, with more or less carbon monoxide, the percentage amount of each varying according to conditions and the process employed. Natural gas consists chiefly of hydrocarbons, almost entirely methane and ethane, and has an average calorific value approximately double that of manufactured city gas. No supply of gaseous hydrocarbons which may be used as a substitute for natural gas is available in large quantities at a low price; neither has there been developed a process for the economical manufacture of a gas which has a calorific value approaching that of natural gas.

The general adaptability of different commonly used and developed gas-making processes to the production of substitute gas has been considered by the Bureau of Mines. It is evident that if any of them are used, when the change is made from natural to manufactured gas having a low calorific value, wholesale adjustments of burners will be necessary. Certain types of appliances will probably have to be replaced by others better adapted for use with manufactured gas or new burners will have to be installed.

In the production of mixed gas of a given calorific value it is essential to consider, besides specific gravity, the composition of the components and of the resulting mixture, since the one has as much effect as the other upon the results attainable upon combustion. Sufficient data for the guidance of gas engineers in producing mixed gases are not yet available. This fact is recognized by the American Gas Association, for which a number of research workers are engaged in testing appliances with gases of various compositions, densities, and calorific values.

A single process having the necessary flexibility of operation without the use of a petroleum product or natural gas has not yet been developed for the production of ordinary city gas. Because the availability of low-priced gas oil will probably diminish as the general supply of natural gas decreases, the interest of the gas industry lies in freeing itself from dependence upon these products for the future production of gas.

The supply of natural gas from a given field decreases

inversely with consumption, but in substantially every instance the quantity available decreases gradually over a period of many years. Therefore, in any scheme devised for the production of substitute gas means should be provided for using the amount of natural gas available during the interim between partial and complete exhaustion. Means have been suggested for accomplishing this result, the general scheme comprising—

First, diluting natural with manufactured gas such as water gas or producer gas during the period when the "pinch" of failing supply is first felt.

Subsequently changing calorific standards to still lower levels, adjusting appliances, and making re-formed from natural gas and mixing it with manufactured gas.

Later, as the supply of natural gas is diminished further, making more manufactured gas, using the natural gas as enricher without cracking (re-forming).

Finally making 100 per cent. substitute gas.

Re-formed gas can be made from natural gas in carbureted water-gas sets with slight modification of the latter. The general process of re-forming gas is not new, but it is believed by the Bureau of Mines that some studies should be made, along with large-scale experiments, in an effort to determine the best methods of operating for different conditions and with different fuels, using substantially standard equipment.

It is known that from water gas can be made synthetic hydrocarbons suitable for use as a carbureting material. This product is not a cheap gas enricher, but it is probable that carburetion of water gas will be cheaper with it than with gas oil in the future. A means is thus at hand for eliminating the uncertainties of supply and price of gas oil in making gas.

Attention has been called to the possibility of storing manufactured gas in exhausted gas wells. Sufficient information is not available to warrant general dependence upon this method of storing gas, but since the initial cost of gas holders is great it seems advisable for companies now producing and distributing natural gas to include in their research programs a study of this phase of the problem as it applies to their particular conditions. The cost of pumping may equal the normal cost of storing in gas holders, but the initial investment for the latter might be largely eliminated.

In the manufactured-gas industry considerable carbureted water gas is being made with coal and coke as generator fuels. As coke is a processed fuel, it is usually more expensive than coal and therefore should not be used for this purpose when a satisfactory market for it can be found. Largely, coke is so used because such a market is lacking. Bituminous coal has been employed with highly satisfactory results, but the economy obtained depends to a great extent upon incorporating some of the blast gas in the "make gas." Because of the relative cost of heat energy in the form of gasified gas oil and straight water gas this procedure is an economic one, but with the price of gas oil rising, other means should be provided for gasifying bituminous coal efficiently—that is, for recovering much of the volatile matter in the finished gas. This can be accomplished with modification of the generating equipment.

The fact is becoming more generally recognized that the sendout load factor has considerable bearing upon the ultimate cost of manufactured gas to consumers. Industrial consumers demand gas at a rate that is more favorable for the economical production and distribution of gas than the rate for domestic consumers; it is partly because of this that gas can be supplied to consumers of the latter class at prevailing prices. Any change in the rate of demand of the different classes of consumers which tends to decrease the fluctuations in demand will also tend to diminish the cost of distributed gas. The ratio of industrial to domestic business is greater in the natural gas than in the manufactured-gas business; with the prospect of losing much of the former upon distributing substitute gas, greater effort should be made to obtain a more favorable load factor than that common to the manufactured-gas industry. Providing special rates—dump rates—to large industrial consumers upon gas burned during periods of light demand might be a step in this direction.

Many gas engineers deplore efforts made to develop a low-temperature carbonization process, basing their views on the fact that high rates of heat transfer and accompanying large throughput per retort per unit of time depend upon maintaining a high differential temperature between the coking mass and the heating surface. Nevertheless, certain important facts that relate to such processes and which are worthy of consideration are—

Gas evolved in processing has much greater calorific value than ordinary coal gas and therefore will "go farther" as an enricher of lean gas than the latter.

The yield of liquid by-products recoverable is much greater than in gas-making processes in common use.

When the char from such a process is briquetted a hard uniform-size fuel of greater density than coke is obtained; these briquets might be more readily salable and to many consumers a better fuel than coke. Incidentally, it is possible to obtain sufficient binder for the briquetting operation from the tar evolved as a by-product in the carbonization of certain coals by the low-temperature process.

The value of briquets and by-products in future may be so great that an industry might be developed, separate from the gas industry, devoted exclusively to the production of these materials.

Steps might well be taken by companies that distribute natural city gas to provide for the capital investment required to maintain a supply of gas when the natural product is no longer available. The means should not only include the accumulation and laying aside of funds for an extensive building program but should be evidenced in present construction work. In California the State fixed the rates for certain gas companies so that it was possible to build and maintain manufactured-gas plants of sufficient size to assure the rendering of good gas service at all times. A precedent has been established which is worthy of study and consideration in connection with rate adjustments in some of our eastern cities now supplied, at least in part, by natural gas.

INCREASING PRODUCTION OF CARBON BLACK FROM NATURAL GAS.

Largely increased yields of carbon black from natural gas have been obtained by injecting the gas into a bed of ignited coke in the course of tests recently conducted by the United States Bureau of Mines, Department of Commerce, coöperating with the Louisville Gas & Electric Company, at Louisville, Kentucky. For many years the better grades of carbon black have been produced by burning hydrocarbon gas incompletely, natural gas in particular, collecting the

free carbon evolved by depositing it upon relatively cool metal surfaces, and also by filtering the gaseous products of combustion through cloth. As a rule, the yield of carbon by this process is less than 1.6 pounds per 1,000 cubic feet of gas treated, whereas it is known that the actual total carbon content of the natural gas averages about 30 to 36 pounds (varying with the composition) per 1,000 cubic feet. Because of the low percentage of the total carbon in the gas extracted by this method, attempts have been made by different investigators to treat natural gas so that a higher yield of carbon would be obtained. Experiments of this nature have not been entirely unsuccessful. It has been found possible to obtain yields of carbon approximating 10 pounds per 1,000 cubic feet of natural gas used, but the carbon thus produced has not graded as highly, for most uses, as that made by the older process. The methods used in obtaining the increased yield consist in causing the natural gas to pass through a chamber containing highly heated solids and to contact the surfaces of the solids. The heating of the solids and the gas-cracking operations are alternated.

In providing the solids, refractory materials are commonly thought of and they have been used. It seems that up to a recent date few attempts have been made to produce carbon black by injecting natural gas into a bed of incandescent solid fuel (coke). In the tests conducted by the Bureau of Mines at Louisville, a standard water-gas generator 9 feet in diameter was used; the operation consisted essentially in first heating the bed of ignited coke by air-blasting, discontinuing this, and then introducing natural gas; the cracked gas with the entrained carbon was removed through the regular gas offtakes. The cycle was repeated regularly. The yield of carbon was approximately 7.0 pounds per 1,000 cubic feet of natural gas passed. The quality of the carbon was not equal to the best but it is believed to be such that it will find use at a fair price in certain industries. Further tests will more definitely establish its value relative to carbons made by other processes.

It is interesting to note that in this process not only can carbon black be recovered, but also a gas containing a high percentage of hydrogen. The hydrogen, in certain localities where natural gas is available, may find use as one of the raw

materials in the synthesis of motor fuel, methanol, ammonia or other materials.

A Report of Investigations more completely describing the experiments will be issued shortly.

EFFECT OF TEMPERATURE UPON THE STRUCTURE OF IRON ORE.

It is not known to what extent iron ores or manganese ores break down into smaller pieces as a result of the temperature changes and mechanical action to which the ore is subjected in the blast furnace. Therefore the relative advantage which some ores may possess by virtue of tending to maintain approximately the same size as when charged can not be appraised. The problem will apply more particularly to ores containing large proportions of finer sizes, but other types also require investigation. If it can be shown that some ores break down a great deal more than others, this characteristic should be taken into account so far as possible in selecting ore mixtures. The problem may have a distinct bearing upon flue-dust losses, a troublesome feature of furnace operation.

The North Central Experiment Station of the United States Bureau of Mines, Minneapolis, Minn., is arranging jointly with the University of Minnesota to secure samples of ore from the mines and in some cases from furnace plants for the purpose of subjecting them to temperature changes and mechanical action that can be duplicated. Tests, similar to the tumbler and shatter tests used for testing the strength of metallurgical coke, may be applied to comparable sizes of various ores that have been subjected to the same temperature changes, or the ore may be heated in a rotary kiln arranged to provide mechanical shock during heating. The advice and coöperation of the industry are solicited in carrying out the investigation.

The Ethics of Multiple Seizure. (*American Journal of Pharmacy*, November, 1929.) "Multiple seizure" is the term applied by the authorities charged with enforcement of the Federal Food and Drug acts, by which samples of suspected products are purchased simultaneously at several points and suits brought promptly at each point. This, of course, involves much greater expense to the marketers of such products than if only one purchase should be made. Complaint has been made by some persons and some organizations about this practice and the authorities have deemed it advisable to publish some facts in regard to the matter. It is pointed out that in many cases if only a single sample is libeled, the docket of the court in that locality may be so overloaded that months may elapse before a trial can be had, thus embarrassing both prosecutor and defendant. In further defense of the practice, the department has set forth in detail the character of some of the articles that have been seized under this system, and it is shown that they represent a most appalling array of fraudulent claims. The number of cases of multiple seizure is not large. From July 1, 1924, to October 1, 1929, about two score instances are enumerated, some of which have not yet been brought to court decision.

A preparation composed mainly of Epsom salt in water with small amounts of other ingredients including iron, quinine and strychnine salts was colored red with a coal-tar dye and labeled as a blood builder, and blood purifier.

It was recommended for free use to very young infants.

A mixture, essentially phosphate, glycerol and water with a small amount of some vegetable extract, and colored with a red dye was represented as a remedy for high blood pressure.

H. L.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, DECEMBER 18, 1929.

THE regular monthly meeting of The Franklin Institute was called to order by the President, Mr. Nathan Hayward, at eight-fifteen p.m. on Wednesday, December eighteenth, 1929, in the Hall of the Institute. The President stated that this was the regular monthly business meeting and called upon the Secretary of the Institute for a statement of the business to be considered.

The Secretary reported that the minutes of the November meeting of the Institute had been printed in full in the December number of the Journal of The Institute, and moved that the minutes of that meeting be approved as printed. The motion was duly seconded and unanimously adopted.

The Secretary reported that since the last meeting the following changes in membership had been made: Seven Resident members and eight Non-Resident members have been elected. To our sincere regret the Institute has learned of the death of a long time member, Mr. D. M. Barringer of Philadelphia.

The Secretary announced that this was the meeting at which nominations must be made for officers and members of the Board of Managers whose terms expire in January. He stated that all names must be in writing and must be signed by at least two members of the Institute.

The following nominations were proposed:

Office	Nominee	Term	Nominated by
President	Mr. Nathan Hayward	One year	Mr. Henry Howson Mr. W. Chattin Wetherill Mr. Marshall S. Morgan
Vice-President	Mr. Walton Forstall	Three years	Mr. Samuel T. Wagner Mr. R. W. Lesley
Treasurer	Mr. Benjamin Franklin	One year	Mr. N. E. Funk Mr. E. W. Sanborn
Board of Managers	Mr. Charles E. Bonine Mr. Edward G. Budd Dr. Walton Clark Mr. Charles Day Mr. Clarence A. Hall Dr. George A. Hoadley Mr. James S. Rogers Mr. Haseltine Smith	Three years	Dr. J. S. Hepburn Dr. Herbert J. Tily Mr. Clark Dillenbeck

The Secretary then announced that other nominations might be offered from the floor, provided they were offered by at least two members of the Institute. He

called for such nominations. None were offered. It was announced that a ballot would be printed and distributed to the membership for the election which will be held at the time of the January meeting of the Institute.

The Secretary made a statement, calling to the attention of the membership and friends of the Institute the Christmas Week Lectures on the James Mapes Dodge Foundation, which will be given on December 26, 27 and 28, in the Hall of the Institute, by Professor Harry N. Holmes, Director of the Department of Chemistry, Oberlin College, on "Colloidal Chemistry." He stated that Professor Holmes was one of the authorities of the country on colloidal chemistry and was a noted lecturer who is distinguished by his ability as an experimentalist. He urged the members and friends of the Institute to consider a ticket of admission as a Christmas gift to young people.

The Secretary reported that certain questions of a financial nature, involving the real estate property of the Institute had arisen which had made it necessary for a special meeting of the Institute to be held in the early future. He therefore requested those members of the Institute who were present to adopt the following resolution:

Resolved: That when this meeting adjourns, it shall adjourn to reconvene at twelve, noon, on Friday, December twenty-seventh.

This motion was offered and properly seconded and was unanimously adopted by the meeting.

The President announced that there was no further business before the meeting and called upon the lecturer of the evening, Professor E. R. Bossange, Director of Architecture of New York University, who spoke delightfully on "Tendencies in Modern Architecture." Professor Bossange reviewed the development of the modern types of architecture and dwelt particularly on essentials of good architecture of all styles and ages. His interesting talk was splendidly illustrated by lantern slides.

After a slight discussion the meeting adjourned, with a rising vote of thanks to Professor Bossange.

HOWARD MCCLENAHAN,
Secretary

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, December 4, 1929.)

HALL OF THE INSTITUTE,
PHILADELPHIA, December 4, 1929.

DOCTOR THOMAS D. COPE *in the Chair.*

The following reports were presented for final action:

No. 2889: Gibson Method of Measuring Liquid Flow in Closed Conduits. This report recommended the award of the Elliott Cresson Medal to Mr. Norman R. Gibson, of Niagara Falls, New York, "In consideration of Mr. Gibson's originality in first adapting a well-known law of mechanics to the measurement of flowing water, his skill and ingenuity in developing apparatus for accurately recording the required data, the simplicity, accuracy and economy of his method, the wide scope of its application in measuring the flow of liquids and its very general adaptation in commercial work."

No. 2914: Photography of Lightning.
This report recommended the award of the John Price Wetherill Medal to Mr. William N. Jennings, of Philadelphia, Pennsylvania, "In consideration of his pioneer work in the photography of flashes of lightning."

The following reports were presented for first reading:
No. 2913: Dubus Pump for Handling Heavy Liquids.
No. 2915 and No. 2916: Franklin Medal.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(*Stated Meeting, Board of Managers, December 18, 1929.*)

RESIDENT.

MR. ARTHUR BLOCH, Merchant, N. Snellenburg and Company, Twelfth and Market Streets, Philadelphia, Pa.

Mr. N. E. BONN, Research Engineer, Rubicon Company, 29 North Sixth Street, Philadelphia, Pa.

DR. NICHOLAS MINORSKY, Assistant Professor of Electrical Engineering, Moore School, University of Pennsylvania, Philadelphia, Pa.

Mr. J. L. Phillips, Assistant Snperintendent, Electric Storage Battery Company, 19th Street and Allegheny Avenue. For mailing, Torresdale, Penna.

MR. HAROLD F. PITCAIRN, Aviation, President Pitcairn-Cierva Autogiro Company of America, Inc., Land Title Building, Philadelphia, Pa.

MR. W. LAWRENCE SAUNDERS, 2d, Medical Publisher, West Washington Square, Philadelphia, Pa. For mailing: Bryn Mawr, Pa.

MR. ALBERT J. WILLIAMS, JR., Electrical Engineer, Leeds and Northrup, 4901 Stenton Avenue, Philadelphia, Pa.

NON-RESIDENT.

Dr. G. L. GILLSON, Geologist, 8060 DuPont Building, Wilmington, Del.

MR. SAMUEL M. KINTNER, Manager of Research, Westinghouse Research Laboratory, East Pittsburgh, Pa.

MR. JOHN A. SEEDE, Electrical Engineer, General Electric Company, Schenectady, N. Y. For mailing: 30 Balltown Road, Schenectady, N. Y.

MR. DING U DOO, Librarian, Nanyang University, Shanghai, China.

CHANGES OF ADDRESS.

MR. W. R. ARMSTRONG, 2108 Poplar Street, Philadelphia, Pa.

MR. ISAAC W. BYERS, 125 North East Fifty-fifth Street, Miami, Florida.

PROFESSOR J. B. CALVA, 124 Drexel Avenue, Lansdowne, Pa.

MR. F. L. DUBOSCUE, 53 Bayview Avenue, Great Neck, L. I., N. Y.

MR. FRANK L. DYER, 24 North Cornwall Avenue, Atlantic City, N. J.

MR. FARNUM F. DORSEY, 5 South Fitzhugh Street, Rochester, New York.

MR. J. HENRY HALLBERG, 112 West Forty-second Street, New York City.

MR. EDWARD HEITMAN, 1403 Longfellow Avenue, Detroit, Michigan.
MR. WALTER G. HENSEL, Hessische Str. 10, bei Hofmann, Berlin, N. 4, Germany.
MR. WILLIAM HURST, 306 Jackson Street, Media, Pa.
MR. FRANK G. KENNEDY, JR., Morris Avenue, Bryn Mawr, Pa.
MR. JAMES LYMAN, 20 North Wacker Drive, Chicago, Illinois.
MR. F. MACMURPHY, 937 East California Street, Pasadena, California.
MR. SIDNEY MASON, The Wellington, 19 & Walnut Streets, Philadelphia, Pa.
MR. W. S. MONROE, 20 North Wacker Drive, Chicago, Illinois.
MR. EDGAR B. NICHOLS, The Brown Instrument Company, Wayne and Roberts Avenue, Philadelphia, Pa.
MR. J. HOWARD PEW, Nineteenth Floor, 1608 Walnut Street, Philadelphia, Pa.
MR. EDGAR S. ROSS, 1811 Franklin Trust Building, Philadelphia, Pa.
MR. H. CLYDE SNOOK, Beechwood Apartments, Summit, N. J.
MR. OSCAR STEINER, 24 East Sixth Street, Media, Pa.
MR. BENEDICT STONE, 230 West 107 Street, New York City.
MR. ALFRED O. TATE, 18 Summerhill Gardens, Toronto, Ontario, Canada
MR. WILLIAM TIDDY, Semet Solvay Engineering Corporation, 40 Rector Street, New York City.

NECROLOGY

Dr. D. M. BARRINGER, Philadelphia, Pa.

LIBRARY NOTES.

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BOOK REVIEWS.

A CHEMICAL DICTIONARY. Based on recent chemical literature. By Ingo W. D. Hackh, College of Physicians and Surgeons, School of Dentistry of San Francisco, California. 790 pages, illustrations, 8vo. Philadelphia, P. Blakiston's Son & Company, Inc., 1929. Price, $10.00.

This is one of the most valuable contributions to chemical literature that has appeared in recent years. In the vast mass of publications, periodicals, government reports, books of all types, pamphlets, many new terms have appeared and many familiar terms have been employed in new senses. To collect from this extensive field such terms, define them and determine, in each case whether the word shall be admitted and if so how much space shall be given to it, is a task that can only be appreciated by those who have had some experience in it. The reviewer feels a sort of special interest in this book in view of the fact that many years ago he talked with the publisher about such an adventure, and indeed began a collection of data. The labor was soon given up. The useful volume, constituting a comprehensive dictionary of chemical terms, is before us, written and edited with great care, printed in excellent form, convenient size, and eminently adapted to the chemical library. The research worker, the commercial analyst, the works-chemist, the teacher, and even the educated man with general interest in this important and widely applicable science will all find the volume invaluable. In the details of the work we find in addition to those formal definitions much tabular matter, diagrammatic representations, explanations of theoretical views, much general information in laboratory operations and considerable matter relating to the industrial applications of the science.

The publication of this book places the English-speaking chemist practically independent of similar works in other languages. Our German co-workers of course excel in such a field but are being followed closely by Americans. One notes for instance very full accounts of the modern theory of atomic structure, details of the more applicable theories of structural formula, lists of abbreviations used in this field and many physical and chemical data. It is worthy to note, to show the scope of the book, a table of spectral classification of stars. The dictionary wanders, therefore, beyond the confines of our own planet.

Testing in various ways the contents of the book we find that it meets every requirement. If it errs at all it is on the side of inclusion, not omission, but this

is a commendable fault. The book is all the better for it. It is indeed a very difficult task to determine what shall be included in a dictionary of science so manifold in its applications as chemistry. In the developments which have been so characteristic of recent years in bio-chemistry, pharmaceutical chemistry and industrial fields, the terms, theories and data have increased so vastly that it has been very difficult for one person to keep apace with the progress. The author of this dictionary has shown unusual versatility in his acquaintance with the science and its accessory phases, and also a clear comprehension of the more abstruse features of modern knowledge.

A critic must find some fault. Much search is required to find fault in this book, but the introduction of portraits seems to the reviewer unwise. They teach nothing and occupy valuable space.

"There's no art
To find the mind's construction in the face."

The long line of seekers for truth to which modern chemistry owes its commanding position are to be honored for their works not for their "counterfeit presentments."

"E'en in their ashes live their wonted fires."

Publisher and printer have done their full share in seconding the work of the author and we have before us, therefore, a volume of convenient size, of comprehensive content, printed in clear type, on good paper, well bound so that its pages lie open at any point and altogether, as said at the beginning of this review, among the most valuable additions to chemical literature of recent years.

HENRY LEFFMANN.

THE CHEMISTRY OF LEATHER MANUFACTURE. By John Arthur Wilson, D. Sc., Chief Chemist, A. F. Gallun & Sons Corporation, Milwaukee, Wisconsin; President, American Leather Chemists' Association. American Chemical Society Monograph Series, Second Edition, Volume 2. 1181 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1929. Price, $10.00.

This volume completes the second edition of this work which is three times the size of the first edition. The leather industry is one of great antiquity and of extensive development. "Nothing like leather" is an old expression, the origin of which seems to be unknown, but standard leathers really deserve some praise for their toughness and durability. The art of tanning was originally conducted purely empirically, probably developed after the long experience and many failures. Modern tanning is conducted on scientific principles as shown by the size of the second volume of this work. The chemistry of leather manufacture is an elaborate science. The transformation of skins into leather involves many complex changes which have become intelligible only since modern developments of physical chemistry and bacteriology. The old methods of tanning by means of vegetable infusions have given way to chemical tanning of which perhaps the chroming methods are the most striking. Three chapters are devoted in this volume to the subject of chrome tanning and the methods. Uses of alum, iron salts and formaldehyde are also described. Full space is devoted to various methods of preparing leather for the market among which we notice a chapter on the patent leather so familiar to the dressy men and women of the midvictorian

age. A chapter on fur is also included and large space is devoted to the physical chemical examinations of leather, detailed attention being given to the methods and apparatus for microscopic examination. The examination and testing of leather and materials used in its production have been extensively developed under the auspices of the American Leather Chemists' Association. There is also a large amount of information concerning the physical properties of manufactured leather especially in reference to porosity and the action of various external agents. The effect of various accessory treatments is also presented.

In short we have in this book a substantial and complete account of the methods of making leather, of preparing it for its various uses and testing the products, and of the materials employed in any part of the manufacture.

HENRY LEFFMANN.

NEW CONCEPTIONS IN COLLOIDAL CHEMISTRY. By Herbert Freundlich, Ph.D., Professor at the Kaiser Wilhelm Institute for Physical Chemistry, Berlin. VII—147 pages, with 47 diagrams and 20 tables, 12mo. New York, E. P. Dutton and Company. Price, $2.00.

"New Conceptions in Colloidal Chemistry" is a compilation of more recently acquired scientific data relating to the nature and structure of colloids and presented by the author in a series of lectures given in the United States during the summer of 1925. Much work has been and still is being done with the object of elaborating the general conceptions of adsorptions, the electric potential of surfaces, coagulation, stability, the state of aggregation and the shape of colloidal particles and such phenomena as photodichroism.

Sufficient experimental work has been done to show that the amount of a substance adsorbed on a surface is in direct relation to its effect upon the lowering of surface tension. By the establishment of such facts the way has been opened for a logical explanation in regard to the action of certain catalytic poisons. A convenient example of such is the poisonous effect exerted by hydrogen cyanide and other substances upon the oxidizing ability of blood charcoal. It is interesting to note that chemiluminescence appears most frequently in heterogeneous systems where adsorption is possible.

More and more it is becoming evident that a study of electrokinetic potentials is necessary if an explanation is to be furnished for coagulations and the diverse valency effects produced by various ions. In the past ten or fifteen years much good work has been done on the determination of electrokinetic potentials between the surface or interface and the interior of a liquid. Studies upon the coagulability of various ions seem to indicate that for dilute solutions there is a correlation between the above effect and their ability to reduce the electrokinetic potentials between the colloidal particles and the dispersing medium. The task of measuring the rate of coagulation has not proven easy, yet sufficient work has been done which points to a possible correlation between the electrokinetic potential of the particles and their mean kinetic energy.

The studies made on the stability of hydrophilic sals have disclosed many interesting facts and the explanations of the effects produced by various coagulating agents appear to be well founded and sound in principle.

Only in the last ten years has any considerable information been obtained which would indicate the form and structure of colloidal particles. This is due in

a large part to the development of the X-ray analysis of the structure of matter. Coindent with X-ray studies, improvements have been made in the technique of ultramicroscopic investigations, particularly in regard to the form and quality of illumination. It is now established that colloidal particles are often crystalline, frequently forming crystals flat or acicular in form. Polarized light is playing an important part in the study of colloidal particles and has been very sucessful in elucidating the structure of the so-called liquid crystals.

Even at the present time, it is found to be almost impossible to produce two exactly similar colloidal solutions of a given stubsance. One of the requirements for achieving duplication would be that the sols be made at exactly the same instant since it is recognized that sols undergo a gradual change as time passes. It is not an easy matter to follow such changes directly and so they are detected more conveniently by allowing the particles to adsorb various dyes or radioactive materials. In the case of some types of sols the changes may be followed with the aid of double refraction measurements.

The general reader will probably find the chapter on Photodichroism and Similar Phenomena to be the most interesting. Photohaloids are sols of light sensitive material, and the silver halides may be taken as the more common examples. These sols possess the remarkable property that after being exposed to polarized light they exhibit both dichroism and double refraction. Dichroism is the ability of the sol to polarize the incident light when illuminated. This phenomenon also has been shown by dyestuffs when dispersed in collodion gels at low concentrations. A parallel is drawn between the action of these dyes and the sensitive retina of the eye. The possible causes for the existence of optically active compounds in nature are reviewed critically and impartially.

The book should prove of interest equally to the colloid chemist and the seeker after general information. To the latter, because a careful summary of the results obtained in the past score years has been presented in a comprehensive and engaging manner; to the former, because each chapter is replete with references to the original articles which thereby bear the recommendation of a most noted scholar of colloid chemistry.

T. K. CLEVELAND.

A TEXT-BOOK OF INORGANIC CHEMISTRY. Edited by J. Newton Friend, D.Sc., Ph.D., F.I.C. Volume 6, Part 3, Vanadium, Niobium and Tantalum, by Sydney Marks, M.Sc., A.I.C. 222 pages, illustrations, 8vo. Philadelphia, J. B. Lippincott Company, 1929. Price, $10.00.

The author remarks in the opening of the general introduction to this series that "during the past few years the civilized world has begun to realize the advantages accruing to scientific research." It is not strictly correct to suggest that the whole world has only awakened to the importance of research for it is evident to all chemists that Germany more than half a century ago realized fully the importance of cultivating theory and practice in coöperation, and the dominance that that country acquired in certain fields of chemical industry was the reward for forethought and earnest application.

The series of volumes on inorganic chemistry, of which this is one, is now too well known to need any special introduction, and the character of the volumes has always been such as to place them in the front rank for information on the subjects covered. The present volume includes three metals which many years ago were

considered extremely rare and known only to the mass of chemists by the few brief allusions in text-books. It is not alone the desire for scientific information that has led entirely to the development of the chemistry of these elements but the industrial importance that they have acquired. As might be expected the bulk of the book is devoted to vanadium that has acquired much importance in certain departments of industrial chemistry. Niobium and tantalum are treated together, though special chapters are given to the alloys and compounds of each respectively. Tantalum, it may be mentioned, has recently acquired an important application as a substitute for platinum in certain forms of electrolytic apparatus. Difficulties that arose in the use of this have been to a great extent solved by a Philadelphia chemist, as will be announced through the proper channels in due course. It remains only to say that in this volume we have a comprehensive and complete study of the chemistry of the three elements with an account of their salts and alloys. There is considerable complication in their compounds and many of these are of interest from a theoretical point of view. The analytical methods required for their detection and determination are also presented.

The series is appearing in comparatively small parts and somewhat irregular as to choice of subjects, but in a work of this scope requiring many editors, it is not possible to get all the lines developed in proper order.

HENRY LEFFMANN.

THE AMAZING BENJAMIN FRANKLIN. Compiled and Edited by J. Henry Smythe, Jr. 296 pages, illustrations, plates, portrait in colors, 12mo. New York, Frederick A. Stokes Company, 1929. Price, $3.00.

"The greatest champions of American Independence were George Washington and Benjamin Franklin." These words open the preface to this volume. It is a rather drastic exclusion, but space would not permit the enumeration of all those who gave great help to the cause. Thomas Paine might have been included which would have added another leaf to Franklin's laurel crown, for he induced Paine to come to America and brought him in contact with the pamphleteers, leading to the writing of our first "best seller," "Common Sense" for eighteen pence, considered as a most powerful agent in developing an independence sentiment.

It is a somewhat favorite theory of present-day biologists that the attributes of humans are wholly due to heredity, the influences of environment being negligible. Yet facts do not seem to justify such positiveness. Gregory VII "greatest of Roman pontiffs and one of the most remarkable men of all times" was of such humble origin that his father's social status is uncertain and his mother's wholly unknown. There does not appear to have been in the forebears of Benjamin Franklin or in his brothers and sisters any tincture of the striking and versatile genius that distinguishes him so clearly in the records of Colonial America. Perhaps it needs but a slight variation in a few brain cells to determine great differences in response to environment. It may be true that

"Great wit to madness nearly is allied
And thin partitions do their bounds divide,"

but Franklin showed no signs of eccentricity so frequently observed in men of genius. He was, as might be expected, now and then at variance with the views of other competent persons. He declined to go a few blocks to see John Fitch's

steamboat, then about to start on its trial trip with several members of the Federal Convention as guests, regarding it as an unpromising experiment, "requiring an extra man to attend to the engine." A curious inconsistency is shown by the fact that he allowed his name to appear on the ticket of the antifederalists at the election for delegates to the State Convention to act upon the Constitution. He sat in the Federal Convention and had signed the document, and it must have been an inadvertence or a peculiar indifference that led to this attitude. It is worth noting, as showing his popularity, that although the antifederalists were overwhelmingly beaten in Philadelphia, he ran far ahead of his colleagues on the ticket.

In a work of this character, essays by many men of many minds, writing at the request of an editor and under certain limitations as to opinions and expression, there is sure to be a lack of spontaneity and careful critical judgment. We are not astonished, therefore, that the articles are of unequal merit, but it will not be necessary to test each by a standard. To the reviewer the least interesting are those on Religion and Humor. Many of our Colonial forefathers cared little for formal creeds or dogmas. The eighteenth century began the era of higher criticism. The discoveries in geology and paleontology shook confidence in the Biblical narrative, and Astruc's "key" opened a yawning gulf of skepticism as to the Mosaic authorship of it. In this fading of the authority of the texts, intelligent and earnest men, such as Voltaire, Jefferson, Paine and Franklin resorted to a spiritualization of Deity and the encouragement of natural religion, talking much about "Nature's God," but their theology was merely a crass deism having no value in either ethics or morals.

In a work of this character we must expect merits to be overdrawn and claims made to which the biographee is not fully entitled. Careful search would doubtless find a number of such, but one is evident and flagrant. On page 264 it is asserted that Franklin was the first to demonstrate the calming effect of oil on waves. Pliny knew the phenomenon and Plutarch gave a correct explanation of it, namely that the oil diminishes the adhesion between the air and water.

Undoubtedly Benjamin Franklin was a remarkable man and we will search through many centuries of human history for his equal and find but few. Aristotle, Julius Cæsar, Roger Bacon, Leonardo da Vinci come into our minds. We must, however, ascribe to environment a large share in Franklin's career. His long life was also an important element. He had a great deal of time in which to do things. Washington died at fifty-seven, Alexander Hamilton at forty-seven, James Wilson at fifty-six. Moreover, conditions in the late Colonial period and early Federal period were eminently favorable for progressive and even revolutionary movements in all departments of human activity. The Atlantic slope was a region of unbounded opportunities. Land was open to settlement on easy terms, the woods were full of game and the streams of fish. Fine harbors were numerous, many of them estuaries of large rivers navigable by deep draft vessels for many miles. Notwithstanding that all the immigrants had come from lands where class consciousness had dominated for centuries accompanied by political and religious oppression, their miscellaneous origins prevented serious stratification in social state or establishment of exclusive dogmas. In fact a dominating purpose in the establishment of the Colonies was economic. The rulers of Europe promoted colonizations for business reasons. The motive was well indicated by the letter that the spiritual adviser, a Jesuit priest, sent to Lord Baltimore concerning the

first company that went out to Maryland in the "Ark" and "Dove," that of the five score or so in the party about twenty were gentlemen and Roman Catholics and the rest mostly artisans and heretics.

After all, real biographies of Washington, Franklin, Jefferson and Paine are still to be written,

"The rank is but the guinea's stamp
The man's the gowd for a' that."

HENRY LEFFMANN.

LA TOPOGRAPHIE SANS TOPOGRAPHES. Traite de Photogrammetrie avec preface de M. Maurice d'Ocagne, Membre de l'Institut. Par F. Ollivier, Chef de Bataillon du Genie, Ingenieur Diplome de l'Ecole Superieure d'Optique. xviii–301 pages, illustrations, 8vo, cloth. Paris, Revue d'Optique Theorique et Instrumentale, 1929.

The geometric relationship of the points on a perspective drawing to the corresponding points which are drawn to a scale of actual dimension on a plan and elevation is familiar to perspective draftsmen and to others who are acquainted with the application of descriptive geometry to perspective. The geometric properties of one perspective drawing or the interrelationship of more than one may accordingly be utilized in determining a plan of the field depicted and the elevation of any points within it; that is, for the construction of a topographical map. A photograph, which is free from appreciable distortion, accordingly furnishes a means of securing perspective pictures for that purpose.

To Laussedat, a French military engineer, is accorded the credit of first having applied and practically developed topographical surveying by means of the camera. The activities of Laussedat began about 1850 and he so thoroughly elaborated suitable methods of procedure that, as late as 1895, the Surveyor General of Dominion Lands, E. Deville says of his contributions: "Wherever photographic surveys are now made, they are executed by the principles laid down by Laussedat."

Capitain Ollivier outlines the evolution of photo-topography or photogrammetry, as he terms it, from pre-Laussedat days, he demonstrates the geometric principles which govern the method and describes early and recent apparatus employed in its application. An account of the survey of Mont-Blanc and environs illustrates convincingly the possibilities of the method for inaccessible localities which cannot be surveyed by customary mehods. One of the serious defects of the photographic method has been the uncertainty of identification of points which have a markedly different aspect on the different negatives. Something radically new was needed to eliminate this fault and it is not surprising that the solution was discovered by a member of the technical staff of the famous firm of Zeiss of Jena.

The inventor of this important advance, which at once eliminates all difficulties of identification, is Doctor Pulfrich of the Zeiss Works. His method consists in the stereoscopic examination of adjacent views of the field. A precise stereoscopic apparatus, the stereocomparator, is employed to bring given pionts into stereoscopic relief. The displacements of index points, which are attached to the moving elements of the adjustment mechanism, determine the coördinates of a point when the stereoscopic effect is attained as viewed through the eye-pieces of

the stereocomparator. The next advance is embodied in the stereoautograph of Lieutenant von Orel, an Austrian officer of The Military Geographical Institute of Vienna. That contribution eliminates the necessity of plotting the readings of the indexes; a tracing point constrained to move in the required relationship to the displacements of the index-points automatically traces contour lines at required elevations. The stereocomparator, it may be mentioned, has recently found important application in map making from airplane photographs. Some notable work of this character in the hands of Mr. L. J. R. Holst is chronicled in this JOURNAL for October 1928.

The author omits little that is needed to supply the reader with the essential elements that enter into the theory of stereogrammetry. A highly detailed account covers the principles of stereoscopic vision, their application to the binocular instruments for the appreciation of relief, and then the geometry of topographical photographs, and their analysis by the stereoscopic comparator. The physiological features of binocular vision are also considered at some length. Adequate illustrated descriptions are included of the phototheodolite, the stereocomparator and the stereoautograph, and the instrumental and operational errors of them are given a thorough examination. Procedure, matters of technique, an examination of results attainable with a comparison with surveys by standard methods, and a consideration of the limitations of photographic methods are dealt with in the final chapters.

The work is a rarely complete and lucid account of the principles, methods and possibilities of modern photographic surveying logically presented in technical language.

L. E. P.

A SOURCE BOOK IN MATHEMATICS. By David Eugene Smith, Ph.D., LL.D., Professor Emeritus in Teachers College, Columbia University, New York City. xvii-701 pages, illustrations. 8vo. cloth. New York, McGraw-Hill Book Company, Inc., 1929. Price, $5.00.

Modern mathematical texts cannot enter very fully into the various historical angles of important theorems, or make more than a brief examination of their origin, the mode of their approach and their early development. The detailed account of these historical theorems is a matter separate and distinct from their presentation in modern and immediately applicable form, and the serious student must turn to original sources for this information. More often than not, the volumes desired are not accessible, and when they are within reach, the reader is confronted with the additional burden of translation. In a conveniently arranged and unusually fine typographical form, Doctor Smith has removed all important difficulties of that sort. The book abounds with a profession of celebrated theorems and other information of mathematical interest from the writings of the great mathematicians which have become history over a period of four centuries closing with the year 1900.

Although, as the author observes, there is no distinct line of demarcation between the different departments of mathematics, he has deemed it advantageous to adopt a rough classification of subject matter. The fields of these divisions are: number; algebra; geometry; probability; and the calculus, functions and quaternions. Such overlapping as may occur in an arbitrary classification is inconse-

quential, especially as a satisfactory index and table of contents are provided. Moreover the topics of these divisions are captioned with the name of the writer followed by the title of the theorem; and the name is easily found in the index. The names of the authors represented naturally appear in several different parts of the volume, sometimes in minor reference, in a footnote or with one or more selected excerpts. In the case of the latter, the page numbers given under the mathematician's name in the index might with advantage have been given in heavier type than those referring to citations. It is a small matter but helpful for quick reference. Each topic is preceded by a short biographical account and information relating to the document and, when translated, the language of the original is mentioned with the name of the translator. Referring to these translations, the author says: "It should be stated that all translations in this work have been contributed without other reward than the satisfaction of assisting students and teachers in knowing the sources of certain phases of mathematics."

As to the choice of subject matter, it is highly representative of what the mathematical student requires, but only actual use is an equitable test of whether the best possible for the intended purpose has been gathered. In the division of Number, besides the purely analytical features, are included early slide-rules, calculating machines, and the modern graphic process of nomography which has become so widely used within comparatively few years. Under Algebra is given a collection of the writings of a dozen or more of its most noted authors. The section on Geometry deals mainly with the use of the modern branches. The division of Calculus, Function Theory and Quaternions is necessarily much condensed, but is consistently representative of the origins of modern processes.

Nowhere else is to be found such an extensive and authoritative collection of abstracts of mathematical classics in convenient form for ready reference. It is safe to say that the work will speedily find a place in every mathematical library.

L. E. P.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 312. The Prediction of Airfoil Characteristics by George J. Higgins. 13 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, ten cents.

This paper describes and develops methods by which the aerodynamic characteristics of an airfoil may be calculated with sufficient accuracy for use in airplane design. These methods for prediction are based on the present aerodynamic theory and on empirical formulas derived from data obtained in the N. A. C. A. variable density wind tunnel at a Reynolds Number corresponding approximately to full scale.

Report No. 317. Wind Tunnel Tests on a Series of Wing Models Through a Large Angle of Attack Range. Part I: Force Tests by Montgomery Knight and Carl J. Wenzinger. 49 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, twenty-five cents.

This investigation covers force tests through a large range of angle of attack on a series of monoplane and biplane wing models. The tests were conducted in the atmospheric wind tunnel of the National Advisory Committee for Aeronautics. The models were arranged in such a manner as to make possible a determination

of the effects of variations in tip shape, aspect ratio, flap setting, stagger, gap, decalage, sweepback, and airfoil profile. The arrangements represented most of the types of wing systems in use on modern airplanes.

The effect of each variable is illustrated by means of groups of curves. In addition, there are included approximate autorotational characteristics in the form of calculated ranges of "rotary instability."

A correction for blocking in this tunnel which applies to monoplanes at large angles of attack has been developed, and is given in an appendix.

Report No. 318. Speed and Deceleration Trials of U. S. S. Los Angeles by S. J. De France and C. P. Burgess. 20 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, ten cents.

The trials reported herein were instigated by the Bureau of Aeronautics of the Navy Department for the purpose of determining accurately the speed and resistance of the *U. S. S. Los Angeles* with and without water recovery apparatus, and to clear up the apparent discrepancies between the speeds attained in service and in the original trials in Germany.

The trials proved very conclusively that the water recovery apparatus increases the resistance about 20 per cent., which is serious, and shows the importance of developing a type of recovery having less resistance.

Between the American and German speed trials without water recovery there remains an unexplained discrepancy of nearly 6 per cent. in speed at a given rate of engine revolutions. Warping of the propeller blades and small cumulative errors of observation seem the most probable causes of the discrepancy.

It was found that the customary resistance coefficients C, are 0.0242 and 0.0293 without and with the water recovery apparatus, respectively. The corresponding values of the propulsive coefficient K, are 56.7 and 44.6. If there is any error in these figures, it is probably in a slight overestimate of C, and an underestimate of K. The maximum errors are almost certainly less than 5 per cent.

No scale effect was detected indicating variation of C with respect to velocity.

Report No. 319. Aerodynamic Characteristics of Twenty-Four Airfoils at High Speeds by L. J. Briggs and H. L. Dryden. 32 pages, illustrations, quarto. Washington, Government Printing Office, 1929.

The aerodynamic characteristics of 24 airfoils are given for speeds of 0.5, 0.65, 0.8, 0.95 and 1.08 times the speed of sound, as measured in an open-jet air stream 2 inches in diameter, using models of 1-inch chord. The 24 airfoils belong to four general groups. The first is the standard R. A. F. family in general use by the Army and Navy for propeller design, the members of the family differing only in thickness. This family is represented by nine members ranging in thickness from 0.04 to 0.20 inch. The second group consists of five members of the Clark Y family, the members of the family again differing only in thickness. The third group, comprising six members, is a second R. A. F. family in which the position of the maximum ordinate is varied. Combined with two members of the first R. A. F. family, this group represents a variation of maximum ordinate position from 30 to 60 per cent. of the chord in two camber ratios, 0.08 and 0.16. The fourth group consists of three geometrical forms, a flat plate, a wedge, and a segment of a right circular cylinder. In addition one section used in the Reed metal propeller was included. These measurements form a part of a general

program outlined at a conference on propeller research organized by the National Advisory Committee for Aeronautics and the work was carried out with the financial assistance of the committee.

Report No. 320. The Measurement of Fluctuations of Air Speed by the Hot-Wire Anemometer by H. L. Dryden and A. M. Kuethe. 26 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, fifteen cents.

The hot-wire anemometer suggests itself as a promising method for measuring the fluctuating air velocities found in turbulent air flow. The only obstacle is the presence of a lag due to the limited energy input which makes even a fairly small wire incapable of following rapid fluctuations with accuracy. This paper gives the theory of the lag and describes an experimental arrangement for compensating for the lag for frequencies up to 100 or more per second when the amplitude of the fluctuation is not too great. An experimental test of the accuracy of compensation and some results obtained with the apparatus in a wind-tunnel air stream are described. While the apparatus is very bulky in its present form, it is believed possible to develop a more portable arrangement.

Report No. 321. Fuel Vapor Pressures and the Relation of Vapor Pressure to the Preparation of Fuel for Combustion in Fuel Injection Engines by Wm. F. Joachim and A. M. Rothrock. 13 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, ten cents.

This investigation on the vapor pressures of fuels was conducted at the Langley Memorial Aeronautical Laboratory at Langley Field, Va., in connection with the general research on combustion in fuel injection engines. The purpose of the investigation was to study the effects of high temperatures such as exist during the first stages of injection on the vapor pressures of several fuels and certain fuel mixtures, and the relation of these vapor pressures to the preparation of the fuel for combustion in high-speed fuel injection engines.

It was found that the vapor pressures of the fuels increased rapidly at high temperatures, the rate of pressure increase becoming greater as the temperatures approached the critical value. Beyond the critical temperature, the rate of pressure increase was constant except at one or more temperatures in the case of certain fuels and their vapors in which chemical changes took place. The chemical changes in some of the fuels were such that the cooling curves were materially different from those obtained during the heating periods. Permanent gases were generated during the heating and cooling periods in the case of the alcohols, gasoline, kerosene, and Diesel engine fuel oil so that the fuels removed from the bomb were materially different from those placed in it. The vapor pressures of the fuels differed from each other considerably, methyl alcohol having the highest vapor pressure, 4370 pounds per square inch, and Diesel engine fuel oil the lowest, 140 pounds per square inch, for the maximum temperature investigated, approximately 850° F.

The data for the vapor pressures of the fuels at high temperatures indicate the compression temperatures required to produce rapid vaporization of the injected fuel for combustion.

Report No. 322. Investigation of Air Flow in Open-Throat Wind Tunnels by Eastman N. Jacobs. 11 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, ten cents.

Tests were conducted on the 6-inch wind tunnel of the National Advisory Committee for Aeronautics during May and June, 1928, to form a part of a research on open-throat wind tunnels. The primary object of this part of the research was to study a type of air pulsation which has been encountered in open-throat tunnels, and to find the most satisfactory means of eliminating such pulsations.

In order to do this it was necessary to study the effects of different variables on all of the important characteristics of the tunnel. This paper gives not only the results of the study of air pulsations and methods of eliminating them, but also the effects of changing the exit-cone diameter and flare and the effects of air leakage from the return passage.

It was found that the air pulsations in the 6-inch wind tunnel could be practically eliminated by using a moderately large flare on the exit cone in conjunction with leakage introduced by cutting holes in the exit cone somewhat aft of its minimum diameter.

Report No. 324. Flight Tests on U. S. S. Los Angeles. Part I: Full Scale Pressure Distribution Investigation by S. J. De France. 33 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, fifteen cents.

The investigation reported herein was conducted by the National Advisory Committee for Aeronautics at the request of and in conjunction with the Bureau of Aeronautics, Navy Department. The purpose was primarily to obtain simultaneous data on the loads and stresses experienced in flight by the *U. S. S. Los Angeles*, which could be used in rigid airship structure design. A secondary object of the investigation was to determine the turning and drag characteristics of the airship. The stress investigation was conducted by the Navy Department.

The aerodynamic loading was obtained by measuring the pressure at 95 locations on the tail surfaces, 54 on the hull, and 5 on the passenger car. These measurements were made during a series of maneuvers consisting of turns and reversals in smooth air and during a cruise in rough air which was just short of squall proportions.

The results of the pressure measurements on the hull indicate that the forces on the forebody of an airship are relatively small. The tail surface measurements show conclusively that the forces caused by gusts are much greater than those caused by horizontal maneuvers. In this investigation the tail surface loadings caused by gusts closely approached the designed loads of the tail structure.

The turning and drag characteristics will be reported in separate papers.

Report No. 325. Flight Tests on *U. S. S. Los Angeles*. Part II: Stress and Strength Determination, by C. P. Burgess. 16 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price fifteen cents.

The tests described in this report furnished data on the actual aerodynamic forces, and the resulting stresses and bending moments in the hull of the *U. S. S. Los Angeles* during as severe still-air maneuvers as the airship would normally be subjected to, and in straight flight during as rough air as is likely to occur in serv-

ice, short of squall or storm conditions. The maximum stresses were found to be within the limits provided for in accepted practice in airship design. Normal flight in rough air was shown to produce forces and stresses about twice as great as the most severe still-air maneuvers. No light was thrown upon the forces which might occur in extreme or exceptional conditions, such as the storm which destroyed the *Shenandoah*.

The transverse aerodynamic forces on the hull proper were found to be small and irregular. Owing to the necessity of conserving helium, it was impossible to fly the airship in a condition of large excess of buoyancy or weight in order to determine the air pressure distribution at a fixed angle of pitch. However, there is every reason to believe that in that condition the forces on the actual airship are as close to the wind-tunnel results as can be determined by present type of pressure measuring apparatus.

It is considered that the most important data obtained are the coefficients of tail-surface forces and hull-bending moments. These are tabulated in this report.

Report No. 326. Tests of Five Metal Model Propellers with Various Pitch Distributions in a Free Wind Stream and in Combination with a Model VE-7 Fuselage, by E. P. Lesley and Elliott G. Reid. 18 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, fifteen cents.

This report describes the tests of five adjustable blade metal model propellers both in a free wind stream and in combination with a model fuselage with stub wings, which were conducted at Stanford University under research authorization of the National Advisory Committee for Aeronautics. The propellers are of the same form and cross section but have variations in radial distributions of pitch. By making a survey of the radial distribution of air velocity through the propeller plane of the model fuselage it is found that this velocity varies from zero at the hub center to approximately free stream velocity at the blade tip.

The tests show that the efficiency of a propeller when operating in the presence of the airplane is, over the working range, generally less than when operating in a free wind stream, but that a propeller with a radial distribution of pitch of the same nature as the radial distribution of air velocity through the propeller plane suffers the smallest loss in efficiency.

Report No. 327. The effect of Supercharger Capacity on Engine and Airplane Performance, by O. W. Schey and W. D. Gove. 20 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price ten cents.

Supercharging has already demonstrated its value as a means of improving the performance of an airplane at moderate and high altitudes. In order to obtain a maximum increase in the performance of an airplane designed to meet definite service requirements, it is necessary that a supercharger of the proper capacity be selected.

The effect of different supercharger capacities on the performance of an airplane and its engine was investigated by the staff of the National Advisory Committee for Aeronautics at Langley Field, Va. The tests were conducted on a DH4-M2 airplane powered with a Liberty 12 engine. In this investigation four

supercharger capacities, obtained by driving a Roots type supercharger at 1.615, 1.957, 2.4, and 3 times engine speed, were used to maintain sea-level pressure at the carburetor to altitudes of 7,000, 11,500, 17,000, and 22,000 feet, respectively.

The performance of the airplane in climb and in level flight was determined for each of the four supercharger drive ratios and for the unsupercharged condition. The engine power was measured during these tests by means of a calibrated propeller.

It was found that very little sacrifice in sea-level performance was experienced with the larger supercharger drive ratios as compared with performance obtained when using the smaller drive ratios.

The results indicate that further increase in supercharger capacity over that obtained when using the 3:1 drive ratio would give a slight increase in ceiling and in high altitude performance, but would considerably impair the performance for an appreciable distance below the critical altitude.

As the supercharger capacity was increased, the height at which sea-level high speeds could be equaled or improved became a larger percentage of the maximum height of operation of the airplane.

Report No. 328. Water Pressure Distribution on a Twin-Float Seaplane, by F. L. Thompson. 18 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price ten cents.

The investigation reported herein was conducted by the National Advisory Committee for Aeronautics at the request of the Bureau of Aeronautics, Navy Department. This is the second of a series of investigations to determine water pressure distribution on various types of seaplane floats and hulls, and was conducted on a twin-float seaplane. It consisted of measuring water pressures and accelerations on a TS-1 seaplane during numerous landing and taxying maneuvers at various speeds and angles.

The results of this investigation show that water pressures as great as 10 lb. per sq. in. may occur at the step in various maneuvers and that pressures of approximately the same magnitude occur at the stern and near the bow in hard pancake landings with the stern well down. At other parts of the float the pressures are less and are usually zero or slightly negative for some distance abaft the step. A maximum negative pressure of 0.87 lb. per sq. in. was measured immediately abaft the step. The maximum positive pressures have a duration of approximately one-twentieth to one one-hundredth second at any given location and are distributed over a very limited area at any particular instant. The greatest accelerations measured normal to the thrust line at the C.G. occurred in pancake landings, and a maximum of 4.3 g. was recorded. Approximate load distribution curves for the worst landing conditions are derived from the data obtained to serve as a guide in static tests.

PUBLICATIONS RECEIVED.

A Source Book in Mathematics, by David Eugene Smith. First edition. 701 pages, illustrations, portraits, 8vo. New York, McGraw-Hill Book Company, 1929. Price $5.00.

Physics of the Home. A textbook for students of home economics by Frederick A. Osborn, Ph.D. Second edition. 397 pages, illustrations, 8vo. New York, McGraw-Hill Book Company, Inc., 1929. Price $3.00.

The Rayon Industry, by Moïs H. Avram B.S., M.E. Second edition. 893 pages, illustrations, plates, 8vo. New York, D. Van Nostrand Company, Inc., 1929. Price $12.00.

Quantum Mechanics, by Edward U. Condon, Ph.D., and Philip M. Morse, Ph.D. 250 pages, illustrations, 8vo. New York, McGraw-Hill Book Company, Inc., 1929. Price $3.00.

Dielectric Phenomena in High-Voltage Engineering, by F. W. Peek, Jr. Third edition, 410 pages, illustrations, 8vo. New York, McGraw-Hill Book Company, Inc., 1929. Price $5.00.

The Amazing Benjamin Franklin. Compiled and edited by J. Henry Smythe Jr. 296 pages, plates, colored portrait, 8vo. New York, Frederick A. Stokes Company, 1929. Price $3.00.

Shift of Civilization. Prepared by Samuel S. Wyer. Chart 24 × 37 inches; Manual describing the chart, 46 pages, illustrations, 8vo. Columbus, Ohio, Fuel-Power-Transportation Educational Foundation, November 1929.

International Telephone and Telegraph Corporation and Associated Companies. Chart showing telephone, telegraph, cable and wireless connections throughout the world. Chart 30 × 62 inches. New York, Information Department, May 1929.

A Yearbook of Railroad Information, 1929 edition. 94 pages, illustrations, 8vo. New York, Committee on Public Relations of the Eastern Railroads, 1929.

National Advisory Committee for Aeronautics. Technical Notes, No. 328. The effect of fuel consumption on cylinder temperatures and performance of a cowled Wright J-5 engine, by Oscar W. Schey. 18 pages, illustrations, quarto. Washington, Committee, 1929.

CURRENT TOPICS.

Building Exits.—A national standard safety code for building exits aimed to cut down the annual loss of 15,000 lives by fire in the United States, has just been approved by the American Standards Association and made available for adoption by state and municipal authorities and for use by architects, engineers and builders. The code was prepared by a technical committee of thirty representatives of safety and insurance organizations, federal government departments, state departments of labor, local fire departments, architects, engineers and others. It represents sixteen years of study by this committee and its predecessors.

The National Fire Protection Association assumed the technical leadership in the preparation of the code under the national standardization procedure of the American Standards Association. The code contains an entirely new section on theatres and other places of public assembly, and a revision and enlargement of a tentative code issued in 1927, covering safe exit provisions for schools, hospitals, department stores, factories and other occupancies.

In the section on theatres, the code says: "The practice of allowing persons to stand near exits, however, should be discouraged if not prohibited altogether. The utilization of standing space within theatres or motion picture theatre auditoriums should be prohibited."

The code also states that: "The line of travel to an exit door by any aisle shall be not greater than 150 feet, and if more than 100 ft. it shall have not more than one angle or turn. Not more than 20 transverse rows of seats shall be placed between cross aisles. Not more than 10 rows of seats nor 12 feet of rise may be placed between cross aisles where steps are provided in the main aisles to overcome differences in level. Cross aisles shall be not less than 44 inches wide, unless railed away from the seats fronting thereon. If so railed the width shall not be less than three feet."

Particular attention in the code is devoted to fire drills.

"In buildings where the population is of a changing character and not under discipline, for example, in hotels or in department stores, no regularly organized fire exit drill, such as that which may be conducted in schools, is possible," says the code. "In such cases the fire exit drills must be limited to the regular employees who, however, can be thoroughly schooled in the proper procedure and

can be trained to properly direct other occupants of the building in case of fire. In occupancies such as hospitals, no regularly constituted fire exit drill is practicable. Here again, however, the regular employees can be rehearsed in the proper procedure in case of fire; such training always is advisable in all occupancies whether or not regular fire exit drills can be held."

In an extensive section of the code devoted to schools, school buildings of low height are recommended because of their greater safety. The code recognizes the necessity for higher buildings in cities, however, and provides accordingly. The code stipulates that schools should have corridors at least eight feet wide and it recommends that in elementary schools lockers should not be located in corridors.

In the part of the code devoted to department stores it is provided that no portion of any building or section shall be more than 100 feet (along the line of travel) from the nearest exit.

R.

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